Exercise 1

(i) Prove that for every integer $k$,

$$f: \mathbb{Z} \to \mathbb{Z}_n, \quad a \mapsto [ka]$$

is a group homomorphism;

(ii) Find all homomorphisms $f: \mathbb{Z}_6 \to S_3$ (where $S_3$ is the symmetric group).

Exercise 2

(i) Prove that a composition of two group homomorphisms is a group homomorphism.

(ii) Prove that homomorphic image of a cyclic group is cyclic.

Exercise 3

Find all homomorphisms:

(i) $f: \mathbb{Z}_2 \to \mathbb{Z}_6$,

(ii) $f: \mathbb{Z}_3 \to \mathbb{Z}_5$. 