November 7, 2016 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2016

Sheet 6

Due: at the end of the tutorial

Exercise 1

Given a binary relation R, we write $a \sim b$ whenever $(a,b) \in R$. For points (x_1, y_1) and (x_2, y_2) in the plane $\mathbb{R}^2$, determine which are equivalence relations:

(i) $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 - y_1 = x_2 - y_2$;

(ii) $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 = x_2$ or $y_1 = y_2$;

(iii) $(x_1, y_1) \sim (x_2, y_2)$ if $y_1 - y_2$ is integer;

(iv) $(x_1, y_1) \sim (x_2, y_2)$ if $x_1 y_2 = x_2 y_1$.

For the equivalence relations, determine the equivalence classes.

Exercise 2

Find all solutions for the equation or prove it has no solutions:

(i) $2 + x \equiv 1 \bmod 8$;

(i) $2x \equiv 5 \bmod 8$;

(iii) $2 + 3x \equiv 1 \bmod 8$.

Exercise 3

Find all subgroups of G:

(i) $G = (\mathbb{Z}_4, +)$;

(ii) $G = (\mathbb{Z}_6, +)$;

(iii) $G = (\mathbb{Z}_8^*, \cdot)$.

Exercise 4

(i) Determine cosets in $\mathbb{Z}_{10}$ of the subgroup $\langle [2] \rangle$ generated by $[2]$.

(ii) Determine left and right cosets in S_3 of the subgroup H generated by the cycle (31).

(iii) If $H \subset G$ is a subgroup, prove that $gH = H$ if and only if $g \in H$.