October 23, 2016 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2016

S h e e t 4

Due: at the end of the tutorial

Exercise 1

Write the permutation as product of disjoint cycles and determine its sign:

(i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 1 & 2 & 4 \end{pmatrix}$;

(ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 5 & 1 & 6 & 2 & 7 & 4 \end{pmatrix}$;

(iii) $(12)(2345)(34567)$ (product of overlapping cycles).

Exercise 2

Find the subgroup generated by the set of permutations (written as cycles):

(i) $\{(124)\}$;

(ii) $\{(12)(345)\}$;

(iii) $\{(12),(34)\}$;

(iv) $\{(12),(234)\}$;

Exercise 3

Which sets of matrices form a group under multiplication:

(i) $\left\{ A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in SL_2(\mathbb{Z}) : a_{11} + a_{22} = \pm 1 \right\}$;

(ii) $\left\{ A \in GL_2(\mathbb{R}) : A \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$;

(iii) $\left\{ A \in O_2(\mathbb{Q}) : A \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$;

(iv) $\left\{ A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \in U_2 : a_{11} = 1 \right\}$.