Exercise 1

Prove or disprove that in any group

(i) identity $e$ is the only solution of the equation $x^2 = x$.

(ii) identity $e$ is the only solution of $x^3 = x$.

Exercise 2

Which groups are cyclic?

(i) the symmetry group $S_2$;
(ii) the symmetry group $S_3$;
(iii) the subgroup $n\mathbb{Z} \subset \mathbb{Z}$;
(iv) the additive group $\mathbb{Z}$;
(v) the group of all translations of $\mathbb{R}$.

Exercise 3

Find the subgroup of $G$ generated by the subset $S$:

(i) $S = \{1, 2\}$ in $G = (\mathbb{Q}, +)$;
(ii) $S = \{-1, 2\}$ in $G = (\mathbb{Q}^*, \cdot)$;
(iii) $S = \{x \in \mathbb{R} : x > 1\}$ in $G = (\mathbb{R}, +)$.

Exercise 4

For $a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}$:

(i) compute $ab$ and $a^{-1}$;
(ii) solve the equation $ax = b$;
(iii) write $b$ as product of transpositions and determine its sign;