October 17, 2016 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2016

S h e e t 3

Due: at the end of the tutorial

Exercise 1

Prove or disprove that in any group

(i) identity e is the only solution of the equation $x^2 = x$.

(ii) identity e is the only solution of $x^3 = x$?

Exercise 2

Which groups are cyclic?

(i) the symmetry group S_2;

(ii) the symmetry group S_3;

(iii) the subgroup $n\mathbb{Z} \subset \mathbb{Z}$;

(iv) the additive group $\mathbb{Z}$;

(v) the group of all translations of $\mathbb{R}$.

Exercise 3

Find the subgroup of G generated by the subset S:

(i) $S = \{1, 2\}$ in $G = (\mathbb{Q}, +)$;

(ii) $S = \{-1, 2\}$ in $G = (\mathbb{Q}^*, \cdot)$;

(iii) $S = \{x \in \mathbb{R} : x > 1\}$ in $G = (\mathbb{R}, +)$.

Exercise 4

For $a = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 5 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}$:

(i) compute ab and a^{-1};

(ii) solve the equation $ax = b$;

(iii) write b as product of transpositions and determine its sign;