October 7, 2016 Lecturer Dmitri Zaitsev

Course 1213 - Introduction to group theory 2016

S h e e t 2

Due: at the end of the tutorial

Exercise 1

Which $*$ define binary operations on the set of integers $\mathbb{Z}$, which are commutative and which are associative:

(i) $m * n = 1$.
(ii) $m * n = m$;
(iii) $m * n = mn - 1$;
(iv) $m * n = \frac{m+n}{2}$;
(v) $m * n = \frac{m+n}{3}$;

Exercise 2

For which binary operations $*$ on the rational numbers $\mathbb{Q}$ there is identity element:

(i) $m * n = mn$;
(ii) $m * n = m + n - 1$;
(iii) $m * n = \frac{m+n}{2}$;
(iv) $m * n = 0$.

Exercise 3

Prove that associativity $(ab)c = a(bc)$ holds automatically whenever one of the elements a, b, c is the identity e.

Exercise 4

Which sets S with given operations are groups:

(i) $S = \{-1, 0, 1\}$ with respect to addition;
(ii) $S = \mathbb{Z} \setminus \{0\}$ with respect to multiplication;
(iii) $S = \{2n : n \in \mathbb{Z}\}$ with respect to addition;
(iv) $S = \{2n : n \in \mathbb{Z}\}$ with respect to multiplication;
(v) $S = \mathbb{Z}$ with respect to subtraction;
(vi) $S = \{2^n : n \in \mathbb{Z}\}$ with respect to multiplication.