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# Exercise sheet 12

**Exercise 47.** *Folding the resolution of the $A_3$ singularity.* Let $X_3 \subset \mathbb{A}^3_{\mathbb{C}}$ be the surface singularity of type $A_3$ with $I(X_3) = \langle x_0^4 - x_1 x_2 \rangle$. Let $C \simeq \mathbb{Z}_2$ be the automorphism group of the Coxeter–Dynkin diagram of type $A_3$, folding $A_3$ to $C_2$.

Show that the generator of the $C$-action on the resolution of $X_3$ is induced by the map

$$(x_0, x_1, x_2) \mapsto (-x_0, x_2, x_1).$$

**Exercise 48.** *Equivalences of unfoldings.* Let $f\colon (\mathbb{C}^{n+1}, 0) \to (\mathbb{C}, 0)$ denote a holomorphic function germ and recall that two unfoldings $F, \widetilde{F}\colon (\mathbb{C}^{n+1} \times \mathbb{C}^k, 0) \to (\mathbb{C}, 0)$ of $f$ are *equivalent* if $\widetilde{F}(z, b) = F(\psi(z, b), b)$ for some holomorphic function germ $\psi\colon (\mathbb{C}^{n+1} \times \mathbb{C}^k, 0) \to (\mathbb{C}^{n+1}, 0)$ with $\psi(z, 0) = z$.

Show that this notion of equivalence indeed defines an equivalence relation between unfoldings of holomorphic function germs. Does this generalize to the notion of "being induced"?

**Exercise 49.** *Jacobi algebras.* Determine a basis of the Jacobi algebra for each of the surface singularities of type $A, D, E$ and observe that for $A_r, D_r, E_r$ the dimension is $r$ in each case.

**Exercise 50.** *A non-simple surface singularity.* Let $\zeta$ be a primitive third root of unity and let $G = \langle \mathrm{diag}(\zeta, \zeta) \rangle \simeq \mathbb{Z}_3$. Consider the affine algebraic variety

$$X = (\mathbb{C}^2/G, \mathbb{C}[z_1, z_2]^G).$$

(a) Show that $X$ is isomorphic to an affine algebraic variety in $\mathbb{A}^4_{\mathbb{C}}$ by finding generators and relations for the coordinate ring $\mathbb{C}[z_1, z_2]^G$.

(b) Show that $X$ has an isolated singularity at 0.