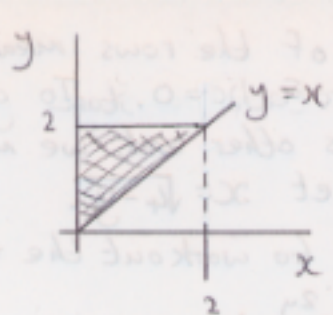




The rows run from $y=x$ to $y=2$, as is indicated by the limits of the inner integral.

Reversing the order gives us rows running in the x -direction from $x=0$ to $x=y$.

So the new integral looks like $\int_0^2 \int_0^y 2y^2 \sin xy \, dx \, dy$

$$\int_0^2 \int_0^y 2y^2 \sin xy \, dx \, dy = \int_0^2 \left[2y^2 \left(-\frac{\cos xy}{y} \right) \right]_0^y \, dy$$

$$= \int_0^2 (-2y \cos y^2 + 2y) \, dy \quad \text{using the substitution } u=y^2, \text{ on the first term, gives}$$

$$\frac{du}{dy} = 2y, \quad dy = \frac{du}{2y}$$

For the limits, when $y=0$, $u=0$ and when $y=2$, $u=4$. we rephrase the

$$\text{integral as } \int_0^4 -2y \cos u \frac{du}{2y} = \int_0^4 -\cos u \, du = [-\sin u]_0^4$$

$$= -\sin 4.$$

The second term, we can integrate conventionally, $\int_0^2 2y \, dy$

$$= [y^2]_0^2 = 4.$$

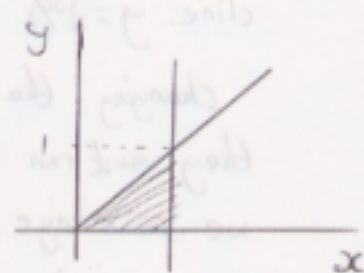
Adding the two terms together gives $4 - \sin 4$.

$$3) \int_0^1 \int_y^1 x^2 e^{xy} \, dx \, dy$$

Solution

Here, the rows run in the x -direction

From the line $x=y$ up to the line $x=1$.



Swapping the direction, we have that the rows run in the y -direction from $y=0$ to the line $y=x$. x then runs from 0 to 1.

$$\int_0^1 \int_0^x x^2 e^{xy} \, dy \, dx = \int_0^1 [x e^{xy}]_0^x \, dx = \int_0^1 x e^{x^2} \, dx$$

$$= \left[\frac{1}{2} e^{x^2} \right]_0^1 = \frac{1}{2}(e-1)$$

$$4) \int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y} \, dy \, dx$$

Solution

The rows of infinitely thin columns run in the y -direction, from $y=0$ up to the line $y=4-x^2$.