the y-direction.

To cover the same area, this means taking an arbitrary value of x and summing up the volumes of a row of infinitely thin columns, starting at the line $y=x$ and finishing at the line $y=b$. This gives us the volume of one row. Then we have to sum up the volumes of all the rows running from $x=0$ up to $x=b$.

So for the inner integration (finding the volume of a row), we have $\int_x^b F(x,y)\,dy$. Including the outer integration (summing up the volume of each row), gives us $\int_0^b \int_x^b F(x,y)\,dy\,dx$.

Now let's have a look at the questions.

Question

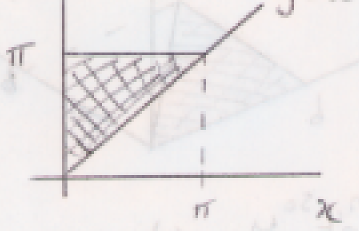
Sketch the region of integration of the following integrals and evaluate them by reversing the order of integration, if necessary.

1) $\int_0^\pi \int_x^\pi \frac{\sin y}{y}\,dy\,dx$.

Solution

y runs from $y=x$ to $y=\pi$. x runs from $x=0$ to $x=\pi$

so the x-y plane looks like below. Because the y-integration is inner most,



the rows run in the y-direction, from $y=x$ to $y=\pi$. Changing the direction of the rows means that x must run from $x=0$ to $x=y$ and that y must run from $y=0$ to $y=\pi$.
So the order-reversed integral is $\int_0^\pi \int_0^y \frac{\sin y}{y}\,dx\,dy$

Evaluating this: $\int_0^\pi \int_0^y \frac{\sin y}{y}\,dx\,dy = \int_0^\pi \left[\frac{x\sin y}{y}\right]_0^y dy = \int_0^\pi \sin y\,dy$

$= \left[-\cos y\right]_0^\pi = 2$.

2) $\int_0^2 \int_x^2 2y^2 \sin xy\,dy\,dx$

Solution

Again, the rows of infinitely thin columns run in the y-direction because the y-integration is inner most. The region over which we are integrating looks like:-