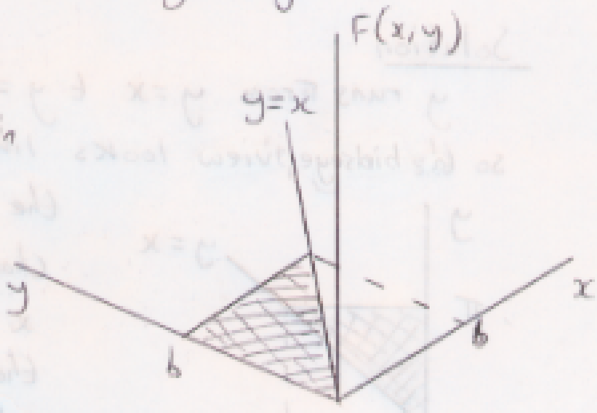
Now the reason that we needed to know this is that we need to be able to manipulate the construction of the integral to solve certain problems.

In particular, some problems are easier to solve, if we can change the order of integration.

If we have an expression that looks like Volume $= \int_0^b \int_0^a F(x,y)\,dx\,dy$, we are free to swap the integral signs and the dx & dy to get Volume $= \int_0^a \int_0^b F(x,y)\,dy\,dx$. However, if any of the limits of the integrals contain equations, instead of numbers, then we have to tread more cautiously. For example, if volume $= \int_0^b \int_0^y F(x,y)\,dx\,dy$ and we try swapping dx & dy and the integral signs, then we end up with volume $= \int_0^y \int_0^b F(x,y)\,dy\,dx$ and instead of getting a number for the volume, we get an equation involving y.

The solution is to look, more carefully, at the shape of the region in the x-y plane over which we are integrating.

In the example vol $= \int_0^b \int_0^y F(x,y)\,dx\,dy$, the region containing the infinitely thin columns is bounded by the limits $x=0$ and $x=y$, in the x-direction and $y=0$ and $y=b$ in the y-direction.



The integral can be interpretted as meaning the following:-

(We start by doing the inner, x, integration, so we pick an arbitrary value of y at which to do it).

- Sum up the volumes of a row of infinitely thin columns from the line $x=0$ to the line $x=y$.

  (This gives us an expression for the volume of each row, now we do the outer integration, in y, which sums up the volumes of the rows.)

- Sum up the volume of each row, starting with the row at $y=0$ and finishing with the row at $y=b$.

What we want to do is reverse the order of integration, so that, instead of the rows running in the x-direction, they run in