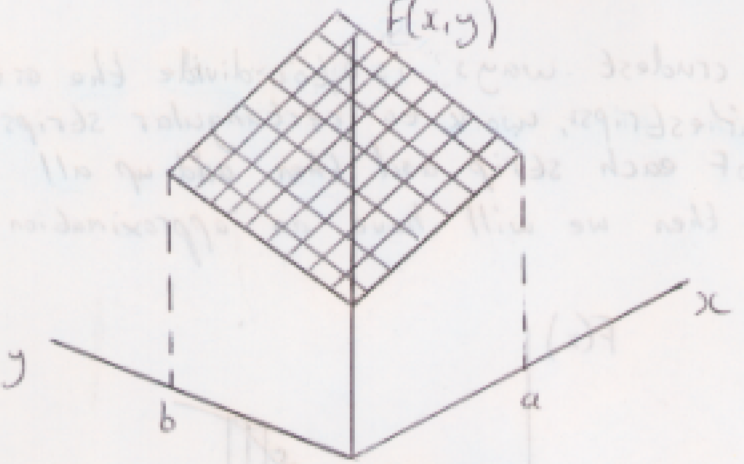
So, instead of $\sum_{x=a}^{b}$ (height of strip centred at x)(width of strip)

we have $\int_a^b F(x)\,dx$.

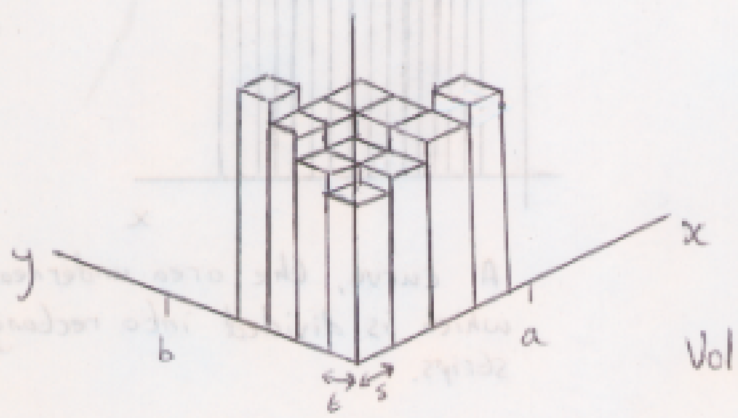
This gives us the relationship between the area under a curve and integration in terms of one variable.

Now let's see what happens when we apply the same strategy to finding the volume beneath a surface.



We shall try to divide the volume into square based columns. The height of a column centred at (x,y) is $F(x,y)$. Its width is s and its depth is t.

So the volume of the column is $F(x,y)st$.



If we sum up all the columns whose bases lie in the region $0 \leq x \leq a$, $0 \leq y \leq b$, then we will have a crude approximation to the volume beneath the surface, in the upper diagram.

$$\text{Volume} = \sum_{y=0}^{b} \sum_{x=0}^{a} F(x,y)\,st$$

As we make the columns thinner and thinner, the approximation will get better and better. In the extreme, where the columns are infinitely thin, the equation becomes exact.

So let's change our notation to reflect that we have gone to this extreme.

s becomes infinitely small and we rename it dx.

t also becomes infinitely small and we rename it dy.

$\sum_{y=0}^{b}$ means "the sum over the columns starting at $y=0$ and finishing at $y=b$." We replace this with $\int_0^b$

Similarly, we replace $\sum_{x=0}^{a}$ with $\int_0^a$

So our expression for the volume is now $\text{Volume} = \int_0^b \int_0^a F(x,y)\,dx\,dy$