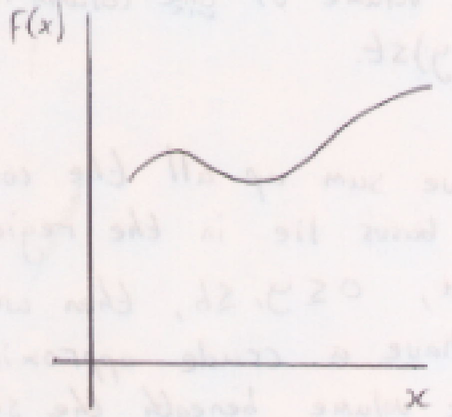
Tutorial 9

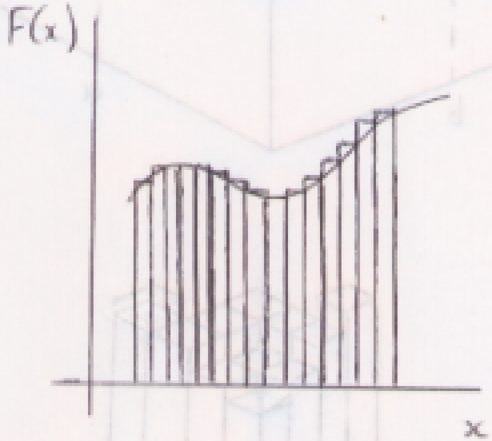
So far, on this course, we have done lots of integration. However, we haven't looked at what integration means. So, before we continue, we shall have a look at how and why integrals are constructed.

Let's start with something that's seemingly unrelated: the area beneath a curve.

One of the easiest and crudest ways to approximate the area under a curve is to divide the area up in to rectangular strips. If we work out the area of each strip and then add up the area of all the strips, then we will have an approximation to the area.



A curve



A curve, the area underneath which is divided into rectangular strips.

$$\text{Area under curve} = \sum (\text{height of strip})(\text{width of strip}).$$

As the strips get thinner and thinner, the approximation gets better and better. In the extreme, where the strips are infinitely thin, the method is no longer an approximation, but is exact.

In this extreme, we also change our notation. The width of each infinitely thin strip is denoted dx and the height of an infinitely thin strip, located at the position $x=a$, is $F(a)$. Instead of using $\sum$ to denote the sum of the strips, we use $\int$. When $\int$ has the limits a and b, ie $\int_a^b$, this is equivalent to $\sum_{x=a}^{b}$, meaning the sum of all the strips, starting with the strip located at $x=a$ and finishing with the strip located at the position $x=b$.