so $400yz^2\underset{\sim}{i} = 2x\lambda\underset{\sim}{i}$

$400xz^2\underset{\sim}{j} = 2y\lambda\underset{\sim}{j}$

$800xyz\underset{\sim}{k} = 2z\lambda\underset{\sim}{k}$

or

① $400yz^2 = 2x\lambda$

② $400xz^2 = 2y\lambda$

③ $800xyz = 2z\lambda$

and $x^2+y^2+z^2-1=0$ ④.

Firstly, we rearrange ① For λ, $\lambda = 200\frac{yz^2}{x}$.

and substitute For λ in ② and ③

②: $400xz^2 = 400\frac{y^2z^2}{x}$ which simplifies to $x = \frac{y^2}{x}$ or $x^2=y^2$

③: $800xyz = 400\frac{yz^3}{x}$ which simplifies to $2x = \frac{z^2}{x}$ or $2x^2=z^2$

substituting For y^2 and z^2 in ④, we have that $x^2+x^2+2x^2-1=0$

which simplifies to $4x^2=1$, $x^2=\frac{1}{4}$ $x=\pm\frac{1}{2}$.

IF $y^2=x^2$, then For each oF the values $x=\frac{1}{2}$ and $x=-1/2$, y can take the value $y=\frac{1}{2}$ or $y=-1/2$.

Similarly, For each oF the Four values oF x and y, $(\frac{1}{2},\frac{1}{2})$, $(\frac{1}{2},-\frac{1}{2})$, $(-\frac{1}{2},\frac{1}{2})$ and $(-\frac{1}{2},-\frac{1}{2})$, $z^2=2x^2$, so $z=\pm\sqrt{2}x$. $z=\pm\frac{1}{\sqrt{2}}$.

This gives us 8 points $(\frac{1}{2},\frac{1}{2},\frac{1}{\sqrt{2}})$, $(\frac{1}{2},\frac{1}{2},\frac{-1}{\sqrt{2}})$, $(\frac{1}{2},\frac{-1}{2},\frac{1}{\sqrt{2}})$, $(\frac{1}{2},\frac{-1}{2},\frac{-1}{\sqrt{2}})$, $(\frac{-1}{2},\frac{1}{2},\frac{1}{\sqrt{2}})$, $(\frac{-1}{2},\frac{1}{2},\frac{-1}{\sqrt{2}})$, $(\frac{-1}{2},\frac{-1}{2},\frac{1}{\sqrt{2}})$, $(\frac{-1}{2},\frac{-1}{2},\frac{-1}{\sqrt{2}})$.

However, the question asks For the highest and lowest temperatures. 4 oF these points ~~give~~ have the same temperature, which is the highest temperature, and the other 4 all share the lowest temperature.

At $(\frac{1}{2},\frac{1}{2},\frac{1}{\sqrt{2}})$, $(\frac{1}{2},\frac{1}{2},\frac{-1}{\sqrt{2}})$, $(\frac{-1}{2},\frac{-1}{2},\frac{1}{\sqrt{2}})$ and $(\frac{-1}{2},\frac{-1}{2},\frac{-1}{\sqrt{2}})$, $T=50$,

whilst at $(\frac{1}{2},\frac{-1}{2},\frac{1}{\sqrt{2}})$, $(\frac{1}{2},\frac{-1}{2},\frac{-1}{\sqrt{2}})$, $(\frac{-1}{2},\frac{1}{2},\frac{1}{\sqrt{2}})$, $(\frac{-1}{2},\frac{1}{2},\frac{-1}{\sqrt{2}})$, $T=-50$.

So the highest and lowest temperatures are 50 and -50, respectively.

4) $F(x,y,z) = x^2+2y-z^2$

$\nabla F = \lambda_1\nabla(2x-y)+\lambda_2\nabla(y+z)$

$\frac{\partial F}{\partial x} = 2x$, $\frac{\partial F}{\partial y} = 2$, $\frac{\partial F}{\partial z} = -2z$