$d = ((x-1)^2+(y-1)^2+(z-1)^2)^{1/2}$. For the sake of ease, we will try to minimize the square of the distance; it will mean that we have fewer indices to worry about and shouldn't change our answer.

$d^2 = (x-1)^2+(y-1)^2+(z-1)^2$. Let's call this s.

$s = d^2$

$\nabla s = \lambda \nabla (x+2y+3z-13)$

$\frac{\partial s}{\partial x}\underline{i} = \lambda \underline{i}, \quad \frac{\partial s}{\partial y}\underline{j} = 2\lambda \underline{j}, \quad \frac{\partial s}{\partial z}\underline{k} = 3\lambda \underline{k}$

$\frac{\partial s}{\partial x} = 2(x-1), \quad \frac{\partial s}{\partial y} = 2(y-1), \quad \frac{\partial s}{\partial z} = 2(z-1)$

So

① $2(x-1) = \lambda$

② $2(y-1) = 2\lambda$

③ $2(z-1) = 3\lambda$

and $x+2y+3z-13=0$ ④.

Using equation ①, we substitute for λ in ② and ③

② becomes $2(y-1) = 4(x-1)$

③ becomes $2(z-1) = 6(x-1)$

Rearranging ②, $2y-2 = 4x-4$, $2y = 4x-2$, $y = 2x-1$.

Rearranging ③, $2z-2 = 6x-6$, $2z = 6x-4$, $z = 3x-2$.

Substitute for y and z in ④ gives

$x + 2(2x-1) + 3(3x-2) - 13 = 0.$

$x + 4x - 2 + 9x - 6 - 13 = 0$

$14x = 21$

$x = \frac{21}{14} = \frac{3}{2}$

Now $y = 2x-1$, so $y = 2\left(\frac{3}{2}\right) - 1 = 3 - 1 = 2$

$z = 3x-2 = 3\left(\frac{3}{2}\right) - 2 = \frac{9}{2} - 2 = \frac{5}{2}$

So our point is $\left(\frac{3}{2}, 2, \frac{5}{2}\right)$

3) $\nabla T = \lambda \nabla (x^2+y^2+z^2-1)$

$\frac{\partial T}{\partial x} = 400yz^2, \quad \frac{\partial T}{\partial y} = 400xz^2, \quad \frac{\partial T}{\partial z} = 800xyz$