At this point, we seem to have a problem. We would like to be able to solve, by simultaneous equation methods, these equations, to get x, y and z. And when you solve simultaneous equations, you need the same number of equations as unknown variables. However, if we include λ, then we have 4 variables with only 3 equations. The solution is that we resurrect our constraint, g. Between the four equations, we can calculate x, y and z and, although we could work it out, at no point do we need to know the value of λ.

We can extend this idea to more than one constraint. Going back to our example of the temperature of a room, suppose that we had a plane that touched or passed through the sphere at some point. We could ask, "What point in the room has the highest temperature, subject to the constraints that the point is on both the plane and the sphere?". This would be the same as asking, "Of all the points where the plane touches the sphere, which has the highest temperature?"

To answer the question, we use two Lagrange multipliers, λ_1 and λ_2.

$\nabla T = \lambda_1 \nabla g_1 + \lambda_2 \nabla g_2$ where g_1 & g_2 are the two constraints.

In terms of the $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$ components

$$\frac{\partial T}{\partial x}\underset{\sim}{i} = \lambda_1 \frac{\partial g_1}{\partial x}\underset{\sim}{i} + \lambda_2 \frac{\partial g_2}{\partial x}\underset{\sim}{i}, \quad \frac{\partial T}{\partial y}\underset{\sim}{j} = \lambda_1 \frac{\partial g_1}{\partial y}\underset{\sim}{j} + \lambda_2 \frac{\partial g_2}{\partial y}\underset{\sim}{j}$$

$$\frac{\partial T}{\partial z}\underset{\sim}{k} = \lambda_1 \frac{\partial g_1}{\partial z}\underset{\sim}{k} + \lambda_2 \frac{\partial g_2}{\partial z}\underset{\sim}{k}$$

or, getting rid of $\underset{\sim}{i}$, $\underset{\sim}{j}$ and $\underset{\sim}{k}$,

$$\frac{\partial T}{\partial x} = \lambda_1 \frac{\partial g_1}{\partial x} + \lambda_2 \frac{\partial g_2}{\partial x}, \quad \frac{\partial T}{\partial y} = \lambda_1 \frac{\partial g_1}{\partial y} + \lambda_2 \frac{\partial g_2}{\partial y}, \quad \frac{\partial T}{\partial z} = \lambda_1 \frac{\partial g_1}{\partial z} + \lambda_2 \frac{\partial g_2}{\partial z}.$$

So now we have 5 equations (these three plus g_1 and g_2, our two constraints) and 5 unknown variables (x, y, z, λ_1 and λ_2). We can solve this, using simultaneous equations methods, as follows :-

Our 5 equations are

(1) $\frac{\partial T}{\partial x} = \lambda_1 \frac{\partial g_1}{\partial x} + \lambda_2 \frac{\partial g_2}{\partial x}$, (2) $\frac{\partial T}{\partial y} = \lambda_1 \frac{\partial g_1}{\partial y} + \lambda_2 \frac{\partial g_2}{\partial y}$, (3) $\frac{\partial T}{\partial z} = \lambda_1 \frac{\partial g_1}{\partial z} + \lambda_2 \frac{\partial g_2}{\partial z}$,

(4) g_1 and (5) g_2