b) If we take $(x-a+1)^{-1}(y-b-2)^{-1}$ and set $c=x-a$ and $d=y-b$, then we have $F(c,d)=(c+1)^{-1}(d-2)^{-1}$. Expanding $(x-a+1)^{-1}(y-b-2)^{-1}$ in powers of $x-a$ and $y-b$, becomes just a question of expanding $(c+1)^{-1}(d-2)^{-1}$ in powers of c and d. This is the same expansion as in the last question, with c replacing x and d replacing y.

we use the result of the last question to get

$$(c+1)^{-1}(d-2)^{-1} = -\tfrac{1}{2}+\tfrac{1}{2}c-\tfrac{1}{4}d-\tfrac{1}{2}c^2-\tfrac{1}{8}d^2+\tfrac{1}{4}cd+\frac{3}{3!}c^2-\frac{3}{(8)(3!)}d^3-\tfrac{1}{2}\tfrac{3}{3!}c^2d+\tfrac{1}{4}\tfrac{3}{3!}dc^2 + \ldots$$

and substitute $x-a$ and $y-b$ back in, for c and d respectively.

$$(x-a+1)^{-1}(y-b-2)^{-1} = -\tfrac{1}{2}+\tfrac{1}{2}(x-a)-\tfrac{1}{4}(y-b)-\tfrac{1}{2}(x-a)^2-\tfrac{1}{8}(y-b)^2+\tfrac{1}{4}(x-a)(y-b)$$
$$+\frac{3}{3!}(x-a)^2-\frac{3}{(8)(3!)}(y-b)^3-\tfrac{1}{2}\tfrac{3}{3!}(x-a)^2(y-b)+\tfrac{1}{4}\tfrac{3}{3!}(y-b)^2(x-a)$$
$$+\ldots$$

c) $F(x,y)=x^7y+x^3-6x^2y^5+y$. $F(0,0)=0$, $\frac{\partial F}{\partial x}=7x^6y+3x^2-12xy^5$

$\left.\frac{\partial F}{\partial x}\right|_{\substack{x=0\\y=0}}=0$, $\frac{\partial F}{\partial y}=x^7-30x^2y^4+1$, $\left.\frac{\partial F}{\partial y}\right|_{\substack{x=0\\y=0}}=1$, $\frac{\partial^2 F}{\partial x^2}=42x^5y+6x-12y^5$

$\left.\frac{\partial^2 F}{\partial x^2}\right|_{\substack{x=0\\y=0}}=0$, $\frac{\partial^2 F}{\partial y^2}=-120x^2y^3$, $\left.\frac{\partial^2 F}{\partial y^2}\right|_{\substack{x=0\\y=0}}=0$, $\frac{\partial^2 F}{\partial x\partial y}=7x^6-60xy^4$,

$\frac{\partial^2 F}{\partial x\partial y}=0$. We could go on, but the important thing to note is that in the definition of the Taylor expansion, there is only one term containing x^7y and only one containing each of x^3, x^2y and y. So, if the Taylor expansion is going to recreate $F(x,y)$, then these terms must be nonzero, whilst all the others are zero.

The only terms that we need to worry about (ie the only ones that will be nonzero) are $\frac{\partial^8 F}{\partial x^7\partial y}$, $\frac{\partial^3 F}{\partial x^3}$, $\frac{\partial^7 F}{\partial x^2\partial y^5}$ and $\frac{\partial F}{\partial y}$.

So $\left.\frac{\partial^8 F}{\partial x^7\partial y}\right|_{\substack{x=0\\y=0}}=7!$, $\frac{\partial^3 F}{\partial x^3}=210x^4y+3!$, $\left.\frac{\partial^3 F}{\partial x^3}\right|_{\substack{x=0\\y=0}}=3!$, $\left.\frac{\partial^7 F}{\partial x^2\partial y^5}\right|_{\substack{x=0\\y=0}}=-12(5!)$,

$\frac{\partial F}{\partial y}=x^7-30x^2y^4+1$, $\left.\frac{\partial F}{\partial y}\right|_{\substack{x=0\\y=0}}=1$

Using this in the Taylor expansion.

$$F(x,y)=\frac{8}{8!}x^7y\left.\frac{\partial^8 F}{\partial x^7\partial y}\right|_{\substack{x=0\\y=0}}+\frac{1}{3!}x^3\left.\frac{\partial^3 F}{\partial x^3}\right|_{\substack{x=0\\y=0}}+\frac{21}{7!}x^2y^5\left.\frac{\partial^7 F}{\partial x^2\partial y^5}\right|_{\substack{x=0\\y=0}}+y\frac{\partial F}{\partial y}$$
$$=\frac{8}{8!}x^7y\,7!+\frac{1}{3!}x^3 3!+\frac{21}{7!}x^2y^5(-12)(5!)+y$$
$$=x^7y+x^3-6x^2y^5+y$$