x and y variables.

Question 1) Using the Formula, Find the line of least squares For the points $(0,1), (1,3), (2,2), (3,4), (4,5)$

Solution

we build a table of all the information that we need.

	x_{exp}	y_{exp}	x_{exp}^2	$x_{exp}y_{exp}$
	0	1	0	0
	1	3	1	3
	2	2	4	4
	3	4	9	12
	4	5	16	20
Σ	10	15	30	39

There are 5 data points, so $n=5$.
From the table $\Sigma x_{exp} = 10$, $\Sigma y_{exp} = 15$, $\Sigma x_{exp}^2 = 30$, $\Sigma x_{exp}y_{exp} = 39$.

so

$$a = \frac{5(39) - (10)(15)}{5(30) - (10)^2}$$

$$= \frac{195 - 150}{150 - 100} = \frac{45}{50} = \frac{9}{10}$$

$$b = \frac{1}{5}\left((15) - \frac{9}{10}(10)\right) = \frac{1}{5}(15-9)$$

$$= \frac{6}{5}.$$

Question 2) a) Find the Taylor expansion of $(x+1)^{-1}(y-2)^{-1}$ in powers of x and y.

b) Find the Taylor expansion of $(x-a+1)^{-1}(y-b-2)^{-1}$ in powers of $x-a$ and $y-b$.

c) Find the Taylor expansion of $x^7y + x^3 - 6x^2y^5 + y$ in powers x and y.

Solution

a) $F(x,y) = (x+1)^{-1}(y-2)^{-1}$, $F(0,0) = -\frac{1}{2}$, $\frac{\partial F}{\partial x} = -(x+1)^{-2}(y-2)^{-1}$, $\left.\frac{\partial F}{\partial x}\right|_{\substack{x=0\\y=0}} = \frac{1}{2}$,

$\frac{\partial F}{\partial y} = -(x+1)^{-1}(y-2)^{-2}$, $\left.\frac{\partial F}{\partial y}\right|_{\substack{x=0\\y=0}} = -\frac{1}{4}$, $\frac{\partial^2 F}{\partial x^2} = 2(x+1)^{-3}(y-2)^{-1}$, $\left.\frac{\partial^2 F}{\partial x^2}\right|_{\substack{x=0\\y=0}} = -1$

$\frac{\partial^2 F}{\partial y^2} = 2(x+1)^{-1}(y-2)^{-3}$, $\left.\frac{\partial^2 F}{\partial y^2}\right|_{\substack{x=0\\y=0}} = -\frac{1}{4}$, $\frac{\partial^2 F}{\partial x \partial y} = (x+1)^{-2}(y-2)^{-2}$, $\left.\frac{\partial^2 F}{\partial x \partial y}\right|_{\substack{x=0\\y=0}} = \frac{1}{4}$

$\frac{\partial^3 F}{\partial x^3} = -6(x+1)^{-4}(y-2)^{-1}$, $\left.\frac{\partial^3 F}{\partial x^3}\right|_{\substack{x=0\\y=0}} = 3$, $\frac{\partial^3 F}{\partial y^3} = -6(x+1)^{-1}(y-2)^{-4}$, $\left.\frac{\partial^3 F}{\partial y^3}\right|_{\substack{x=0\\y=0}} = \frac{-6}{16} = -\frac{3}{8}$

$\frac{\partial^3 F}{\partial^2 x \partial y} = -2(x+1)^{-3}(y-2)^{-2}$, $\left.\frac{\partial^3 F}{\partial^2 x \partial y}\right|_{\substack{x=0\\y=0}} = -\frac{1}{2}$, $\frac{\partial^3 F}{\partial x \partial y^2} = -2(x+1)^{-2}(y-2)^{-3}$, $\left.\frac{\partial^3 F}{\partial x \partial y^2}\right|_{\substack{x=0\\y=0}} = \frac{1}{4}$

so $(x+1)^{-1}(y-2)^{-1} = -\frac{1}{2} + \frac{1}{2}x - \frac{1}{4}y - \frac{1}{2}x^2 - \frac{1}{8}y^2 + \frac{1}{4}xy + \frac{3}{3!}x^3 - \frac{3}{(3)3!}y^3 - \frac{1}{2}\frac{3}{3!}x^2y + \frac{1}{4}\frac{3}{3!}y^2x + \ldots$