At $(-\frac{2}{3},\frac{2}{3})$ $\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right)-\left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 = 12$ $\therefore$ $(-\frac{2}{3},\frac{2}{3})$ is either a maximum or a minimum.

$\frac{\partial^2 F}{\partial x^2} = 6x$. At $(-\frac{2}{3},\frac{2}{3})$, $\frac{\partial^2 F}{\partial x^2} = -4$. Therefore, $(-\frac{2}{3},\frac{2}{3})$ is a maximum.

III) $F(x,y) = 3y^2 - 2y^3 - 3x^2 + 6xy$ $\quad \frac{\partial F}{\partial x} = -6x + 6y$ $\quad \frac{\partial F}{\partial y} = 6y - 6y^2 + 6x$

$-6x + 6y = 0 \Rightarrow 6x = 6y \Rightarrow x = y$. Substitute for x in the next line

$6y - 6y^2 + 6x = 0. \Rightarrow 6y - 6y^2 + 6y = 0 \Rightarrow 12y - 6y^2 = 0 \Rightarrow y(12-6y) = 0.$

Either $y=0$ or 2. $x=y$ so if $y=0, x=0$ and if $y=2, x=2$.

This gives two points $(2,2)$ and $(0,0)$

$\frac{\partial^2 F}{\partial x^2} = -6$ $\quad \frac{\partial^2 F}{\partial y^2} = 6 - 12y$ $\quad \frac{\partial^2 F}{\partial x \partial y} = 6$

$\left(\frac{\partial^2 F}{\partial x^2}\right)\left(\frac{\partial^2 F}{\partial y^2}\right)-\left(\frac{\partial^2 F}{\partial x \partial y}\right)^2 = -6(6-12y) - 36 = -36 + 72y - 36 = 72y - 72.$

At $(0,0)$ $72y - 72 = -72$. $\therefore$ $(0,0)$ is saddle point.

At $(2,2)$ $72y - 72 = 72$. $\therefore$ $(2,2)$ is either a maximum or a minimum.

$\frac{\partial^2 F}{\partial x^2} = -6$. $\therefore$ $(2,2)$ is a maximum.

PS A last note on Taylor expansions.

To find the coefficient (ie the number) that goes with a differentiation in the Taylor expansion, we need the 'choose' function, ${}^nC_r$.

$${}^nC_r = \frac{n!}{r!(n-r)!}, \text{ where } 0! = 1$$

So, for a term, in the Taylor expansion, that contains $\frac{\partial^{a+b} F}{\partial x^a \partial y^b}$, the number that goes with it is $\frac{{}^{a+b}C_a}{(a+b)!} = \frac{1}{(a+b)!}\frac{(a+b)!}{a!b!} = \frac{1}{a!b!}$.