$$\left.\frac{\partial^3 F}{\partial d^3}\right|_{\substack{c=0\\d=0}} = -6(1-c)^{-1}(1-d)^{-4}(-1)\Big|_{\substack{c=0\\d=0}} = 6,\quad \left.\frac{\partial^3 F}{\partial^2 c\,\partial d}\right|_{\substack{c=0\\d=0}} = 2(1-c)^{-3}(-1)(1-d)^{-2}(-1)\Big|_{\substack{c=0\\d=0}}$$

$$= 2,\quad \left.\frac{\partial^3 F}{\partial c\,\partial d^2}\right|_{\substack{c=0\\d=0}} = 2(1-c)^{-2}(-1)(-1)(1-d)^{-3}\Big|_{\substack{c=0\\d=0}} = 2$$

So the series looks like this

$$F(c,d) = 1 + c + d + \tfrac{1}{2}c^2 2 + \tfrac{1}{2}d^2 2 + cd + \tfrac{1}{3!}c^3 6 + \tfrac{1}{3!}d^3 6 + \tfrac{3}{3!}c^2 d\, 2 + \tfrac{3}{3!}d^2 c\, 2 + \ldots$$

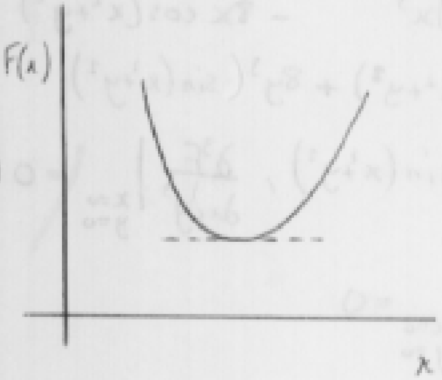
substituting for c & d, we have

$$F(x,y) = 1 + (x-a) + (y-b) + (x-a)^2 + (y-b)^2 + (x-a)(y-b) + (x-a)^3 + (y-b)^3 + (x-a)^2(y-b) + (x-a)(y-b)^2 + \ldots$$

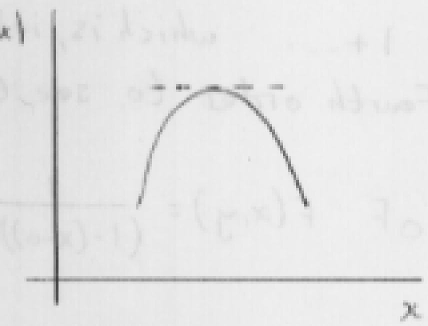
For the Final question, consider the plot of an equation. At the lowest point on the curve, the gradient is zero (A). Similarly, at the highest point on the curve, the gradient is zero (B).

So, if we want to find the maximum or minimum point, we differentiate the equation, set it equal to zero and solve the equation for the appropriate value of x.

(A)

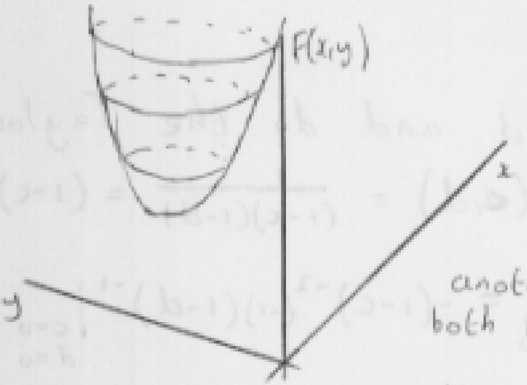


(B)

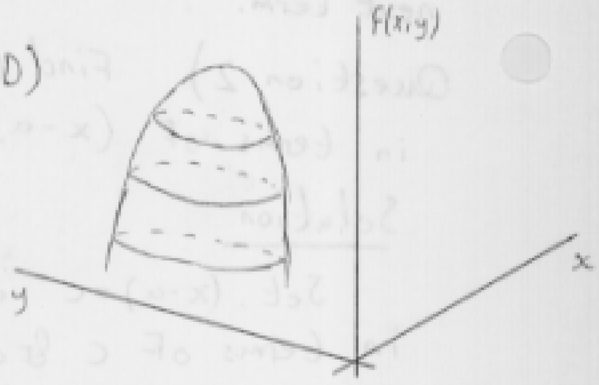


We can extend this idea to two variables. At the minimum point in two dimensions, (C), the gradient is zero in both the x direction ~~of~~ and the y direction. Similarly, at the maximum (D) the gradient is zero in both directions.

(C)

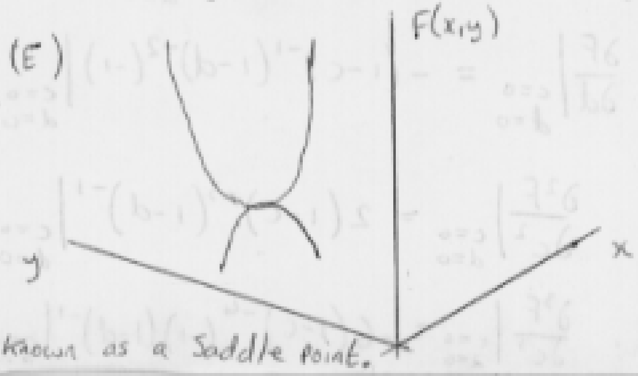


(D)



However, when we have two variables, there is another shape of function that has a point where both the gradients in the x and y direction are zero.

(E)



In one direction, the shape is a maximum. In the other, it is a minimum. These two directions are perpendicular.
The shape is very similar to ~~very~~ ~~similar~~ the part of a saddle that you sit on. For that reason, the point where both gradients are zero is known as a Saddle Point.