so $F(x,y) = 0 + 0 + 0 + 0 + xy + 0 + 0 + 0 + 0 + 0 + \ldots$
$= xy + \ldots$

The second way, we just need to expand $\sin x$

$F = \sin x \quad F(0) = 0, \quad \frac{dF}{dx}\Big|_{x=0} = 1, \quad \frac{d^2F}{dx^2}\Big|_{x=0} = 0, \quad \frac{d^3F}{dx^3}\Big|_{x=0} = -1$

so $F(x,y) = y\left(0 + x - \frac{1}{3!}x^3 + \ldots\right) = yx - \frac{1}{3!}yx^3 + \ldots$

III) $F = \cos(x^2+y^2) \qquad F(0,0) = 1 \qquad \frac{\partial F}{\partial x} = -2x\sin(x^2+y^2)$

$\frac{\partial F}{\partial x}\Big|_{\substack{x=0\\y=0}} = 0, \quad \frac{\partial F}{\partial y} = -2y\sin(x^2+y^2), \quad \frac{\partial F}{\partial y}\Big|_{\substack{x=0\\y=0}} = 0, \quad \frac{\partial^2 F}{\partial x^2} = -4x^2\cos(x^2+y^2) - 2\sin(x^2+y^2)$

$\frac{\partial^2 F}{\partial x^2}\Big|_{\substack{x=0\\y=0}} = 0, \quad \frac{\partial^2 F}{\partial y^2} = -2\sin(x^2+y^2) - 4y^2\cos(x^2+y^2), \quad \frac{\partial^2 F}{\partial y^2}\Big|_{\substack{x=0\\y=0}} = 0$

$\frac{\partial^2 F}{\partial x \partial y} = -4xy\cos(x^2+y^2), \quad \frac{\partial^2 F}{\partial x \partial y}\Big|_{\substack{x=0\\y=0}} = 0, \quad \frac{\partial^3 F}{\partial x^3} = -4x\cos(x^2+y^2) + 8x^3\sin(x^2+y^2) - 8x\cos(x^2+y^2)$

$\frac{\partial^3 F}{\partial x^3}\Big|_{\substack{x=0\\y=0}} = 0, \quad \frac{\partial^3 F}{\partial y^3} = -4y\cos(x^2+y^2) - 8y\cos(x^2+y^2) + 8y^3\sin(x^2+y^2)$

$\frac{\partial^3 F}{\partial y^3}\Big|_{\substack{x=0\\y=0}} = 0, \quad \frac{\partial^3 F}{\partial x \partial y^2} = -4x\cos(x^2+y^2) + 8y^2x\sin(x^2+y^2), \quad \frac{\partial^3 F}{\partial x \partial y^2}\Big|_{\substack{x=0\\y=0}} = 0$

$\frac{\partial^3 F}{\partial x^2 \partial y} = +8x^2y\sin(x^2+y^2) - 4y\cos(x^2+y^2), \quad \frac{\partial^3 F}{\partial x^2 \partial y}\Big|_{\substack{x=0\\y=0}} = 0$

This seems to give us $\cos(x^2+y^2) = 1 + \ldots$ which is, in fact, correct. We would have to go to fourth order to see the next term.

Question 2) Find the Taylor series of $F(x,y) = \frac{1}{(1-(x-a))(1-(y-b))}$ in terms of $(x-a)$ & $(y-b)$.

Solution

Set $(x-a) = c$ and $(y-b) = d$ and do the Taylor series in terms of c & d $\quad F(c,d) = \frac{1}{(1-c)(1-d)} = (1-c)^{-1}(1-d)^{-1}$

$F(0,0) = \frac{1}{(1-0)(1-0)} = 1, \quad \frac{\partial F}{\partial c}\Big|_{\substack{c=0\\d=0}} = -(1-c)^{-2}(-1)(1-d)^{-1}\Big|_{\substack{c=0\\d=0}}$

$= 1, \quad \frac{\partial F}{\partial d}\Big|_{\substack{c=0\\d=0}} = -(1-c)^{-1}(1-d)^{-2}(-1)\Big|_{\substack{c=0\\d=0}} = 1, \quad \frac{\partial^2 F}{\partial d^2}\Big|_{\substack{c=0\\d=0}} = 2(1-c)^{-1}(1-d)^{-3}\Big|_{\substack{c=0\\d=0}}$

$= 2, \quad \frac{\partial^2 F}{\partial c^2}\Big|_{\substack{c=0\\d=0}} = 2(1-c)^{-3}(1-d)^{-1}\Big|_{\substack{c=0\\d=0}} = 2, \quad \frac{\partial^2 F}{\partial c \partial d}\Big|_{\substack{c=0\\d=0}} = (1-c)^{-2}(1-d)^{-2}\Big|_{\substack{c=0\\d=0}}$

$= 1, \quad \frac{\partial^3 F}{\partial c^3}\Big|_{\substack{c=0\\d=0}} = -6(1-c)^{-4}(-1)(1-d)^{-1}\Big|_{\substack{c=0\\d=0}} = 6$