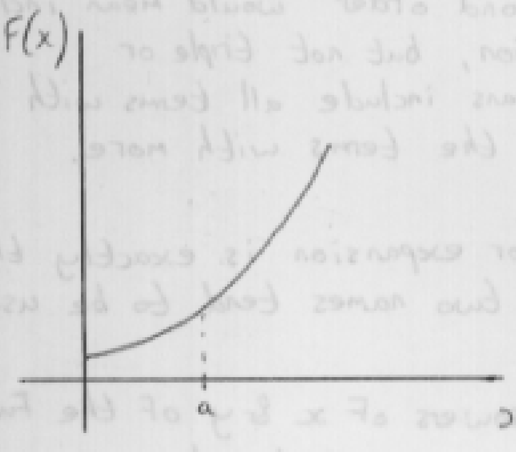
Tutorial 6



The Taylor expansion is a way of expressing a function at a, F(a), using only information obtained at x=0. It is an infinite series.

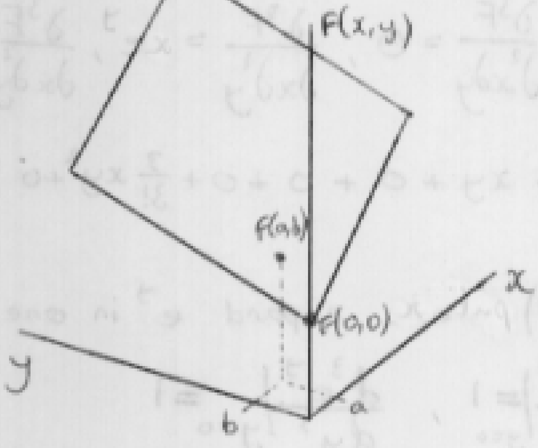
$$F(a) = F(0) + a\frac{dF}{dx}\Big|_{x=0} + \frac{1}{2!}a^2\frac{d^2F}{dx^2}\Big|_{x=0}$$

$$+ \frac{1}{3!}a^3\frac{d^3F}{dx^3}\Big|_{x=0} + \frac{1}{4!}a^4\frac{d^4F}{dx^4}\Big|_{x=0} + \frac{1}{5!}a^5\frac{d^5F}{dx^5}\Big|_{x=0}$$

$+ \ldots$ and so on.

When there are infinitely many terms on the right hand side, the expression is exact. Otherwise it is an approximation. However the more terms that there are, on the right hand side, the better the approximation.

$\frac{dF}{dx}\Big|_{x=0}$ means differentiate F with respect to x and then substitute in $x=0$. Similarly, $\frac{d^2F}{dx^2}\Big|_{x=0}$ means differentiate F twice and then substitute $x=0$.



When F is a function of two variables, x and y, the Taylor expansion looks slightly different

$$F(a,b) = F(0,0) + a\frac{\partial F}{\partial x}\Big|_{\substack{x=0\\y=0}} + b\frac{\partial F}{\partial y}\Big|_{\substack{x=0\\y=0}}$$

$$+ \frac{1}{2!}a^2\frac{\partial^2F}{\partial x^2}\Big|_{\substack{x=0\\y=0}} + ab\frac{\partial^2F}{\partial x\partial y}\Big|_{\substack{x=0\\y=0}} + \frac{1}{2!}b^2\frac{\partial^2F}{\partial y^2}\Big|_{\substack{x=0\\y=0}}$$

$$+ \frac{1}{3!}a^3\frac{\partial^3F}{\partial x^3}\Big|_{\substack{x=0\\y=0}} + \frac{3}{3!}a^2b\frac{\partial^3F}{\partial^2x\partial y}\Big|_{\substack{x=0\\y=0}} + \frac{3}{3!}ab^2\frac{\partial^3F}{\partial x\partial y^2}\Big|_{\substack{x=0\\y=0}}$$

$$+ \frac{1}{3!}b^3\frac{\partial^3F}{\partial y^2}\Big|_{\substack{x=0\\y=0}} + \ldots$$

The expression $\frac{\partial^2F}{\partial x\partial y}$ means differentiate F x with respect to either x or y and then take the expression that you get and differentiate with respect to the other variable. It doesn't matter which order you do it in.