solution

I) $F = x^2+y^2+z$ $\quad \frac{\partial F}{\partial x} = 2x \quad \frac{\partial F}{\partial y} = 2y \quad \frac{\partial F}{\partial z} = 1$

At $(1,2,4)$ $\frac{\partial F}{\partial x} = 2, \frac{\partial F}{\partial y} = 4 \quad \frac{\partial F}{\partial z} = 1$ $\quad$ so Normal vector $= 2\underset{\sim}{i} + 4\underset{\sim}{j} + \underset{\sim}{k}$

To get tangent plane $\begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-2 \\ z-4 \end{pmatrix} = 0$

$2(x-1)+4(y-2)+1(z-4) = 2x-2+4y-8+z-4=0$

$2x+4y+z = 14$ is the equation of the tangent plane

II) $\cancel{\ }$ $z = x\cos y - ye^x$, so $F = z - x\cos y + ye^x$

$\frac{\partial F}{\partial x} = -\cos y + ye^x \quad \frac{\partial F}{\partial y} = x\sin y + e^x \quad \frac{\partial F}{\partial z} = 1$

At $(0,0,0)$ $\frac{\partial F}{\partial x} = -1 \quad \frac{\partial F}{\partial y} = 1 \quad \frac{\partial F}{\partial z} = 1.$ Normal vector $= -\underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$

tangent plane: $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x-0 \\ y-0 \\ z-0 \end{pmatrix} = 0$

$-x+y+z=0$

III) $x^2+y^2+z^2=3$ at $(1,1,1)$. $\quad F = x^2+y^2+z^2$

$\frac{\partial F}{\partial x} = 2x \quad \frac{\partial F}{\partial y} = 2y \quad \frac{\partial F}{\partial z} = 2z.$ At $(1,1,1)$ $\quad \frac{\partial F}{\partial x} = 2 \quad \frac{\partial F}{\partial y} = 2, \quad \frac{\partial F}{\partial z} = 2$

Normal vector is $2\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$

tangent plane $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-1 \\ z-1 \end{pmatrix} = 0$ $\qquad 2(x-1)+2(y-1)+2(z-1)=0$

$2x+2y+2z=6$

$x+y+z=3.$

IV) $x+y+z=1$ at $(0,1,0)$ $\quad \frac{\partial F}{\partial x} = 1 \quad \frac{\partial F}{\partial y} = 1 \quad \frac{\partial F}{\partial z} = 1$

Normal vector $= \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$

tangent plane: $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x-0 \\ y-1 \\ z-0 \end{pmatrix} = 0$ $\qquad x+(y-1)+z=0$

$x+y+z=1$

- equation of tangent plane.