To get the equation of the line, we use $y-y_1 = m(x-x_1)$
$y-(-1) = \frac{1}{2}(x-2)$, $y+1 = \frac{1}{2}x-1$, $y = \frac{1}{2}x-2$.

To get the equation of the tangent plane, instead of a tangent line, we first work out the normal (or gradient) vector. The normal vector will be 3 dimensional, because tangent planes only arise in 3dimensions.

A vector on the tangent plane will be perpendicular to the normal vector, so we take the 'dot' product between them. and find out what the equation of the tangent plane must be, to make the 'dot' product zero.

For escample.

$F = x^2+2y^2+2z^2$ at $(1,2,1)$. At $(1,2,1)$ $F = 11$

To get the normal vector, $\frac{\partial F}{\partial x} = 2x$, $\frac{\partial F}{\partial y} = 4y$ $\frac{\partial F}{\partial z} = 4z$

At $(1,2,1)$ $\frac{\partial F}{\partial x} = 2$ $\frac{\partial F}{\partial y} = 8$ $\frac{\partial F}{\partial z} = 4$ So the normal vector is $2\underline{i} + 8\underline{j} + 4\underline{k}$.

To get the tangent plane we calculate the equation of z from

$$\begin{pmatrix} 2 \\ 8 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x-x_1 \\ y-y_1 \\ z-z_1 \end{pmatrix} = 0$$

where x_1 is the x coordinate of the point and y_1 and z_1 are the y and z coordinates.

From the example above $\begin{pmatrix} 2 \\ 8 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-2 \\ z-1 \end{pmatrix} = 0$

$$\begin{pmatrix} 2 \\ 8 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-2 \\ z-1 \end{pmatrix} = 2(x-1) + 8(y-2) + 4(z-1)$$

$$= 2x-2+8y-16+4z-4 = 0$$

so $2x+8y+4z = 22$ is the equation of the tangent plane.

Question 2) Find the normal vector and the equation of the tangent plane to the surfaces.

I) $x^2+y^2+z = 9$ at $(1,2,4)$

II) $z = x\cos y - ye^x$ at $(0,0,0)$

III) $x^2+y^2+z^2 = 3$ at $(1,1,1)$

IV) $x+y+z = 1$ at $(0,1,0)$