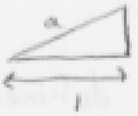
When we looked at the cylinder several tutorials ago, we saw that $dV = \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial h}dh$, ie the change in the volume can be expressed in terms of the change in the radius and height of the cylinder.

Similarly $d(8) = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy = 2x\,dx + 2y\,dy$. However, 8 is a constant number and when we differentiate it, it goes to zero. (F is constant at 8, because we are only interested in values of x & y on the level curve of height 8). $0 = 2x\,dy + 2y\,dy$. At (2,2), this is $0 = 4dy + 4dx$

Rearranging, $-4dy = 4dx$. $\frac{dy}{dx} = -\frac{4}{4} = -1$.

Now, to get a vector of gradient -1, we have to look at what $\frac{dy}{dx}$ means. $\frac{dy}{dx}$ is the change in the y direction divided by the change in the x direction, so 

the line a has gradient 1, because for 1 we move in the x direction, the line a moves 1 in the y direction. So if we move 1 in the x direction, a line of gradient -1 moves -1 in the y direction, so the vector is $\underset{\sim}{i} - \underset{\sim}{j}$.

Now, to get the equation of a straight line, we use the formula $y - y_1 = m(x - x_1)$, where m is the gradient and x_1 and y_1 are 2 and 2, because we're using the point (2,2)

So $y - 2 = -1(x-2)$, $y - 2 = -x + 2$, $y = -x + 4$ is the equation of the tangent line.

Question 1) Find the gradient vector, tangent vector and the equation of the tangent line of the ellipse $\frac{x^2}{4} + y^2 = 2$, at $(2,-1)$.

<u>Solution.</u> $F = \frac{x^2}{4} + y^2$ is our function and at $(2,-1)$ $F = 2$.

The gradient vector is $\frac{\partial F}{\partial x}\underset{\sim}{i} + \frac{\partial F}{\partial y}\underset{\sim}{j} = \frac{2x}{4}\underset{\sim}{i} + 2y\underset{\sim}{j}$.

At $(2,-1)$ this is $\underset{\sim}{i} - 2\underset{\sim}{j}$.

Now to get the tangent vector, we differentiate $2 = \frac{x^2}{4} + y^2$

$0 = \frac{2x}{4}dx + 2y\,dy$ At $(2,-1)$ this is $dx - 2dy = 0$

Rearranging, we get $\frac{dy}{dx} = \frac{1}{2}$, so the tangent vector is $\underset{\sim}{i} + \frac{1}{2}\underset{\sim}{j}$