$$= \frac{-3}{(x^2+y^2+z^2)^{3/2}} + \frac{3}{(x^2+y^2+z^2)^{3/2}} = 0$$

6) $F = e^{3x+4y}\cos 5z$. $\quad \frac{\partial F}{\partial x} = 3e^{3x+4y}\cos 5z$ (By chain rule)

$$\frac{\partial F}{\partial y} = 4e^{3x+4y}\cos 5z \qquad \frac{\partial F}{\partial z} = -5e^{3x+4y}\sin 5z$$

$$\frac{\partial^2 F}{\partial x^2} = 9e^{3x+4y}\cos 5z \qquad \frac{\partial^2 F}{\partial y^2} = 16e^{3x+4y}\cos 5z \qquad \frac{\partial^2 F}{\partial z^2} = -25e^{3x+4y}\cos 5z$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = 9e^{3x+4y}\cos 5z + 16e^{3x+4y}\cos 5z - 25e^{3x+4y}\cos 5z = 0.$$

Which values of r in $F(x,y,z) = (x^2+y^2+z^2)^r$ satisfy Laplace?

$$\frac{\partial F}{\partial x} = r(x^2+y^2+z^2)^{r-1}2x. \qquad \frac{\partial^2 F}{\partial x^2} = r(r-1)(x^2+y^2+z^2)^{r-2}4x^2 + r(x^2+y^2+z^2)^{r-1}2$$

(By product and chain rule)

$$\frac{\partial F}{\partial y} = r(x^2+y^2+z^2)^{r-1}2y \qquad \frac{\partial^2 F}{\partial y^2} = r(r-1)(x^2+y^2+z^2)^{r-2}4y^2 + r(x^2+y^2+z^2)^{r-1}2$$

$$\frac{\partial F}{\partial z} = r(x^2+y^2+z^2)^{r-1}2z \qquad \frac{\partial^2 F}{\partial z^2} = r(r-1)(x^2+y^2+z^2)^{r-2}4z^2 + r(x^2+y^2+z^2)^{r-1}2$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = r(r-1)(x^2+y^2+z^2)^{r-2}4x^2 + r(x^2+y^2+z^2)^{r-1}2 + r(r-1)(x^2+y^2+z^2)^{r-2}4y^2$$
$$+ r(x^2+y^2+z^2)^{r-1}2 + r(r-1)(x^2+y^2+z^2)^{r-2}4z^2 + r(x^2+y^2+z^2)^{r-1}2$$

$$= (4x^2+4y^2+4z^2)\,r(r-1)(x^2+y^2+z^2)^{r-2} + 6r(x^2+y^2+z^2)^{r-1}$$

To satisfy Laplace, all this must be equal to zero, so the question becomes, "For what value of r does this equal zero?"

$$(4x^2+4y^2+4z^2)\,r(r-1)(x^2+y^2+z^2)^{r-2} + 6r(x^2+y^2+z^2)^{r-1} = 0$$

$$4(x^2+y^2+z^2)\,r(r-1)(x^2+y^2+z^2)^{r-2} + 6r(x^2+y^2+z^2)^{r-1} = 0$$

$$4r(r-1)(x^2+y^2+z^2)^{r-1} + 6r(x^2+y^2+z^2)^{r-1} = 0$$

$$4r(r-1) + 6r = 0 \qquad 4r^2 - 4r + 6r = 0 \qquad 4r^2 + 2r = 0$$

$$2r^2 + r = 0 \qquad r(2r+1) = 0$$

so r either $= 0$ or $-1/2$ for $r(2r+1)$ to equal zero.