

$$3) F = e^{-2y} \cos 2x. \quad \frac{\partial F}{\partial x} = -2e^{-2y} \sin 2x \quad \frac{\partial^2 F}{\partial x^2} = -4e^{-2y} \cos 2x$$

$$\frac{\partial F}{\partial y} = -2e^{-2y} \cos 2x \quad \frac{\partial^2 F}{\partial y^2} = 4e^{-2y} \cos 2x \quad \frac{\partial F}{\partial z} = 0$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = 0 = -4e^{-2y} \cos 2x + 4e^{-2y} \cos 2x = 0$$

$$4) F = \log[(x^2+y^2)^{1/2}]. \quad \frac{\partial F}{\partial x} = \frac{\frac{1}{2}(x^2+y^2)^{-1/2} 2x}{(x^2+y^2)^{1/2}} = \frac{x}{(x^2+y^2)} \quad \left(\text{Using chain rule as well as } \frac{d}{dx} \ln A = \frac{\frac{dA}{dx}}{A} \right)$$

$$\frac{\partial F}{\partial y} = \frac{\frac{1}{2}(x^2+y^2)^{-1/2} 2y}{(x^2+y^2)^{1/2}} = \frac{y}{(x^2+y^2)} \quad \frac{\partial^2 F}{\partial x^2} = \frac{1}{(x^2+y^2)} - \frac{2x^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 F}{\partial y^2} = \frac{1}{x^2+y^2} - \frac{2y^2}{(x^2+y^2)^2} \quad \frac{\partial F}{\partial z} = 0$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = \frac{1}{x^2+y^2} - \frac{2x^2}{(x^2+y^2)^2} + \frac{1}{x^2+y^2} - \frac{2y^2}{(x^2+y^2)^2} = \frac{2}{x^2+y^2} - \frac{2x^2+2y^2}{(x^2+y^2)^2} \\ = \frac{2}{x^2+y^2} - \frac{2}{x^2+y^2} = 0.$$

$$5) F = (x^2+y^2+z^2)^{-1/2}. \quad \frac{\partial F}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} 2x \quad (\text{By chain rule})$$

$$\frac{\partial^2 F}{\partial x^2} = -(x^2+y^2+z^2)^{-3/2} - x(x^2+y^2+z^2)^{-5/2} \left(-\frac{3}{2}\right) 2x \quad (\text{By product rule}) \\ = \frac{-1}{(x^2+y^2+z^2)^{3/2}} + \frac{3x^2}{(x^2+y^2+z^2)^{5/2}}$$

$$\frac{\partial F}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} 2y. \quad \frac{\partial^2 F}{\partial y^2} = -(x^2+y^2+z^2)^{-3/2} - y(x^2+y^2+z^2)^{-5/2} \left(-\frac{3}{2}\right) 2y \\ = \frac{-1}{(x^2+y^2+z^2)^{3/2}} + \frac{3y^2}{(x^2+y^2+z^2)^{5/2}}$$

$$\frac{\partial F}{\partial z} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2} 2z. \quad \frac{\partial^2 F}{\partial z^2} = -(x^2+y^2+z^2)^{-3/2} - z(x^2+y^2+z^2)^{-5/2} \left(-\frac{3}{2}\right) 2z \\ = \frac{-1}{(x^2+y^2+z^2)^{3/2}} + \frac{3z^2}{(x^2+y^2+z^2)^{5/2}}$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 F}{\partial z^2} = \frac{-1}{(x^2+y^2+z^2)^{3/2}} + \frac{3x^2}{(x^2+y^2+z^2)^{5/2}} + \frac{-1}{(x^2+y^2+z^2)^{3/2}} + \frac{3y^2}{(x^2+y^2+z^2)^{5/2}}$$

$$\frac{-1}{(x^2+y^2+z^2)^{3/2}} + \frac{3z^2}{(x^2+y^2+z^2)^{5/2}} = \frac{-3}{(x^2+y^2+z^2)^{3/2}} + \frac{3(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{5/2}}$$