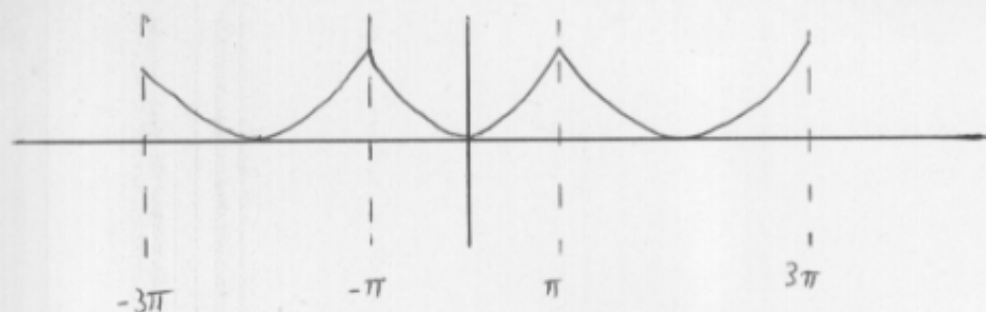




A Century of Controversy Over the Foundations of Mathematics

So anyway you can see this is fantastic stuff! I don't know whether it's mathematics, but it's very imaginative, it's very pretty, and actually there was a lot of practical spin-off for pure mathematicians from what Cantor was doing.

Some people regarded set theory as a disease. Poincaré, the great French mathematician, said set theory is a disease, he said, from which I hope future generations will recover. But other people rebid all of mathematics using the set-theoretic approach. So modern topology and a lot of abstract mathematics of the twentieth century is a result of this more abstract set-theoretic approach, which generalised questions. The mathematics of the nineteenth century was at a lower level in some ways, more involved with special cases and formulas. The mathematics of the twentieth century — it's hard to write a history of mathematics from the year ten-thousand looking back because we're right here — but the mathematics of the twentieth century you could almost say is set-theoretical, "structural", would be a way to describe it. The mathematics of the nineteenth century was concerned with formulas, infinite Taylor series perhaps. But the mathematics of the twentieth century went on to a set-theoretic level of abstraction.

And in part that's due to Cantor, and some people hate it saying that Cantor wrecked and ruined mathematics by taking it from being concrete and making it wispy-wispy, for example, from hard analysis to abstract analysis. Other people loved this. It was very controversial. It was very controversial, and what didn't help is in fact that there were some contradictions. It became more than just a matter of opinion. There were some cases in which you got into really bad trouble, you got obvious nonsense out. And the place you get obvious nonsense out in fact is a theorem of Cantor's that says that for any infinite set there's a larger infinite set which is the set of all its subsets, which sounds pretty reasonable. This is Cantor's diagonal argument — I don't have time to give you the details.

So then the problem is that if you believe that for any infinite set there's a set that's even larger, what happens if you apply this to the universal set, the set of everything? The problem is that by definition the set of everything has everything, and this method supposedly would give you a larger set, which is the set of all subsets of everything. So there's got to be a problem, and the problem was noticed by Bertrand