

$$\text{II) } a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 dx = \frac{1}{\pi} \left[\frac{1}{3} x^3 \right]_{-\pi}^{\pi} = \frac{1}{\pi} \left[\frac{1}{3} \pi^3 + \frac{1}{3} \pi^3 \right] = \frac{2}{3} \pi^2$$

$$\begin{aligned} a_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \cos kx dx = \frac{1}{\pi} \left(\left[\frac{x^2}{k} \sin kx \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{2x}{k} \sin kx dx \right) \\ &= \frac{1}{\pi} \left(\frac{\pi^2}{k} \sin k\pi - \frac{\pi^2}{k} \sin(-k\pi) - \left[\frac{-2x}{k^2} \cos kx \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{2}{k^2} \cos kx dx \right) \\ &= \frac{1}{\pi} \left(0 - 0 + \frac{2\pi}{k^2} \cos k\pi + \frac{2\pi}{k^2} \cos(-k\pi) - \left[\frac{2}{k^3} \sin kx \right]_{-\pi}^{\pi} \right) \\ &= \frac{1}{\pi} \left(\frac{4\pi}{k^2} \cos k\pi - \frac{2}{k^3} \sin k\pi + \frac{2}{k^3} \sin(-k\pi) \right) \\ &= \frac{4}{k^2} \cos k\pi = \frac{4}{k^2} (-1)^k \end{aligned}$$

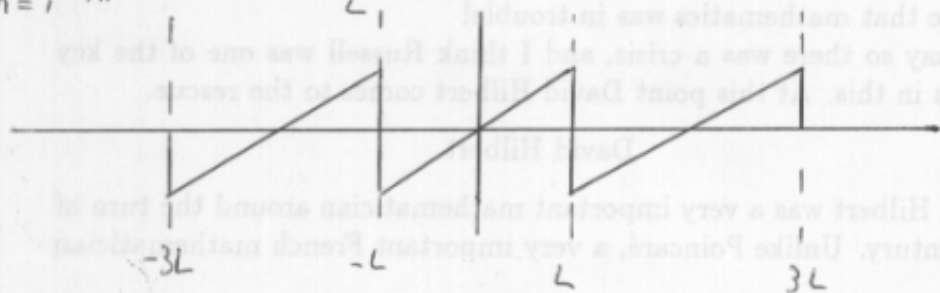
$$\begin{aligned} b_k &= \frac{1}{\pi} \int_{-\pi}^{\pi} x^2 \sin kx dx = \frac{1}{\pi} \left(\left[-\frac{x^2}{k} \cos kx \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} -\frac{2x}{k} \cos kx dx \right) \\ &= \frac{1}{\pi} \left(-\frac{\pi^2}{k} \cos k\pi + \frac{\pi^2}{k} \cos(-k\pi) + \left[\frac{2x}{k^2} \sin kx \right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} \frac{2}{k^2} \sin kx dx \right) \\ &= \frac{1}{\pi} \left(-\frac{\pi^2}{k} \cos(k\pi) + \frac{\pi^2}{k} \cos(k\pi) + \frac{2\pi}{k^2} \sin k\pi + \frac{2\pi}{k^2} \sin(-k\pi) + \left[\frac{2}{k^3} \cos kx \right]_{-\pi}^{\pi} \right) \\ &= \frac{1}{\pi} \left(0 + 0 + 0 + 0 + \frac{2}{k^3} \cos(k\pi) - \frac{2}{k^3} \cos(-k\pi) \right) \\ &= \frac{1}{\pi} \left(\frac{2}{k^3} \cos(k\pi) - \frac{2}{k^3} \cos(k\pi) \right) = 0. \end{aligned}$$

$$\text{So } F(x) = \frac{1}{3} \pi^3 + \sum_{n=1}^{\infty} \frac{4}{n^2} (-1)^n \cos nx.$$

Note that in (II) we have returned to the previous definition of the Fourier series $(F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx)$ because the interval, on which we are given $F(x)$, is $[-\pi, \pi]$.

In the interval $[-3L, 3L]$, the Fourier series of (I) looks exactly the same as $F(x)$, only repeated three times and with the ends joined up.

$$\sum_{n=1}^{\infty} \frac{2L}{n\pi} (-1)^{n+1} \sin \frac{n\pi x}{L} \text{ looks like}$$



In the interval $[-3\pi, 3\pi]$, the Fourier series of (II) looks like