Tutorial 20

In the last tutorial, we saw that any periodic function, $F(x)$, can be represented as an infinite series of sin functions, cosine functions plus a constant.

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

If the function is defined on the interval $[-\pi, \pi]$, then the coefficients can be calculated with the formulae

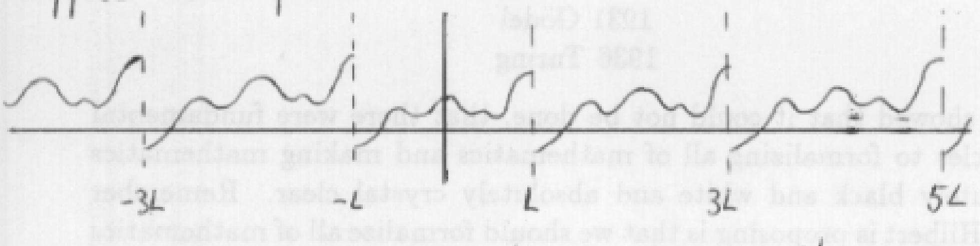
$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx\,dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \sin nx\,dx$$

All this has assumed that the functions have been periodic with a period of 2π, ie that the function is the same, in the interval $[\pi, 3\pi]$ as it is in the interval $[-\pi, \pi]$. However, this doesn't have to be the case. We can have a function that is still periodic, but is periodic over a different interval. When this is the case, we have to modify our Fourier series, so that it, too, is periodic over a different interval.

Suppose our periodic function is defined in the region $[-L, L]$



We can still represent the function with a Fourier series. However, we have to modify the sin and cosine functions so that they have the same periodicity as the function.

When, in the previous tutorial, the function was defined in the interval $[-\pi, \pi]$, $\cos Kx$ had the value 1 when $x = 0$ and $(-1)^K$ when $x = \pm\pi$. On the interval $[-L, L]$, we want $\cos Kx$ to be equal to 1 when $x = 0$ and equal to $(-1)^K$ when $x = \pm L$. We can do this by replacing $\cos(Kx)$ with $\cos(K\frac{\pi x}{L})$. Similarly we can replace $\sin(kx)$ with $\sin(\frac{K\pi x}{L})$. By making this substitution, it has the effect of stretching the sin and cosine functions to fit the new interval.