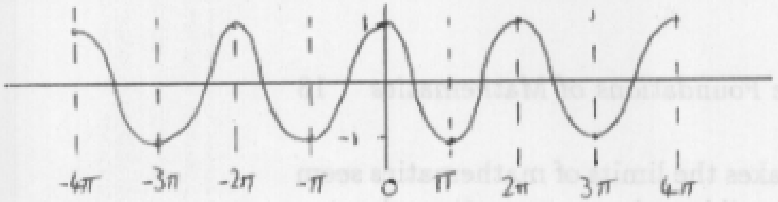
$$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin kx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} \sin kx\,dx + \frac{1}{\pi}\int_{0}^{\pi} -\sin kx\,dx$$

$$= \frac{1}{\pi}\left[-\frac{\cos kx}{k}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{\cos kx}{k}\right]_{0}^{\pi} = \frac{1}{k\pi}\Big(-\cos 0 + \cos(-k\pi) + \cos(k\pi) - \cos 0\Big)$$

$\cos 0 = 1$, $\cos(-k\pi) = \cos k\pi$ and $\cos k\pi = (-1)^k$. We can obtain $\cos k\pi = (-1)^k$ from a plot of $\cos kx$



$\cos 0 = 1$
$\cos \pi = -1$
$\cos 2\pi = 1$
$\cos 3\pi = -1$
$\cos 4\pi = 1$

Hence $\cos n\pi = (-1)^n$

So $b_k = \frac{1}{k\pi}\left(-2 + 2(-1)^k\right) = \frac{2}{k\pi}\left((-1)^k - 1\right) \leftarrow$ the 'general' term.

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left((-1)^n - 1\right)\sin kx$$

II) $f(x) = |x|$ is the same as $f(x) = x \quad 0 < x < \pi$
$\qquad = -x \quad -\pi < x < 0$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} -x\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\,dx = \frac{1}{\pi}\left[-\frac{1}{2}x^2\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{1}{2}x^2\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left(\frac{1}{2}\pi^2 + \frac{1}{2}\pi^2\right) = \pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} -x\cos nx\,dx + \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,dx$$

$$= \frac{1}{\pi}\left(\left[-\frac{x}{n}\sin nx\right]_{-\pi}^{0} - \int_{-\pi}^{0} -\frac{1}{n}\sin nx\,dx\right) + \frac{1}{\pi}\left(\left[\frac{x}{n}\sin nx\right]_{0}^{\pi} - \int_{0}^{\pi}\frac{1}{n}\sin nx\,dx\right)$$

$$= \frac{1}{\pi}\left(0 - \frac{\pi}{n}\sin(-n\pi) - \left[\frac{1}{n^2}\cos nx\right]_{-\pi}^{0}\right) + \frac{1}{\pi}\left(\left(\frac{\pi}{n}\sin(n\pi) - 0\right) - \left[\frac{-1}{n^2}\cos nx\right]_{0}^{\pi}\right)$$

where we have used integration by parts.

$\sin(n\pi) = 0 \qquad \cos n\pi = \cos(-n\pi) = (-1)^n \qquad \cos(0) = 1 \qquad \sin(0) = 0$

$$a_n = \frac{1}{\pi}\left(-\frac{1}{n^2} + \frac{1}{n^2}(-1)^n + \frac{1}{n^2}(-1)^n - \frac{1}{n^2}\right) = \frac{1}{\pi n^2}\left(2(-1)^n - 2\right) = \frac{2}{n^2\pi}\left((-1)^n - 1\right)$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{0} -x\sin nx\,dx + \frac{1}{\pi}\int x\sin nx\,dx$$

$$= \frac{1}{\pi}\left(\left[\frac{x}{n}\cos nx\right]_{-\pi}^{0} - \int_{-\pi}^{0}\frac{1}{n}\cos nx\,dx + \left[-\frac{x}{n}\cos nx\right]_{0}^{\pi} - \int_{0}^{\pi} -\frac{1}{n}\cos nx\,dx\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi}{n}\cos(-n\pi) - \left[\frac{1}{n^2}\sin nx\right]_{-\pi}^{0} - \frac{\pi}{n}\cos(n\pi) + \left[\frac{1}{n^2}\sin nx\right]_{0}^{\pi}\right)$$

$$= \frac{1}{\pi}\left(\frac{\pi}{n}\cos(n\pi) - \frac{\pi}{n}\cos(n\pi)\right) = 0 \qquad b_n = 0$$