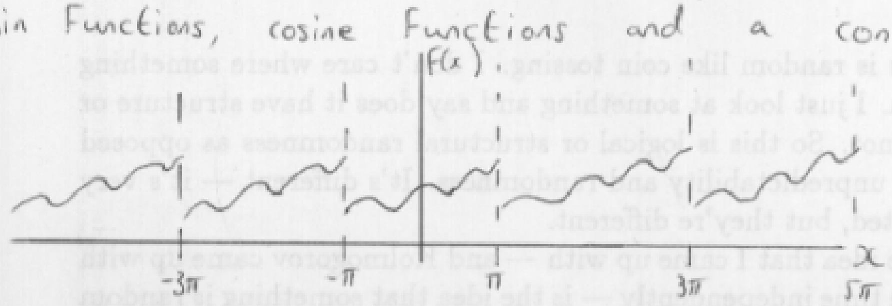
Fourier Series

Jean Baptiste Joseph Fourier was a French mathematician who, whilst serving in Napoleon's army, developed a number of methods for analysing functions.

The key to Fourier's analysis was the idea that any function that a periodic (meaning that it looks the same in the intervals $[-\pi, \pi]$, $[\pi, 3\pi]$, $[3\pi, 5\pi]$ etc) can be represented by an infinite series of sin functions, cosine functions and a constant.



← An example of a periodic function

So for any periodic function $F(x)$

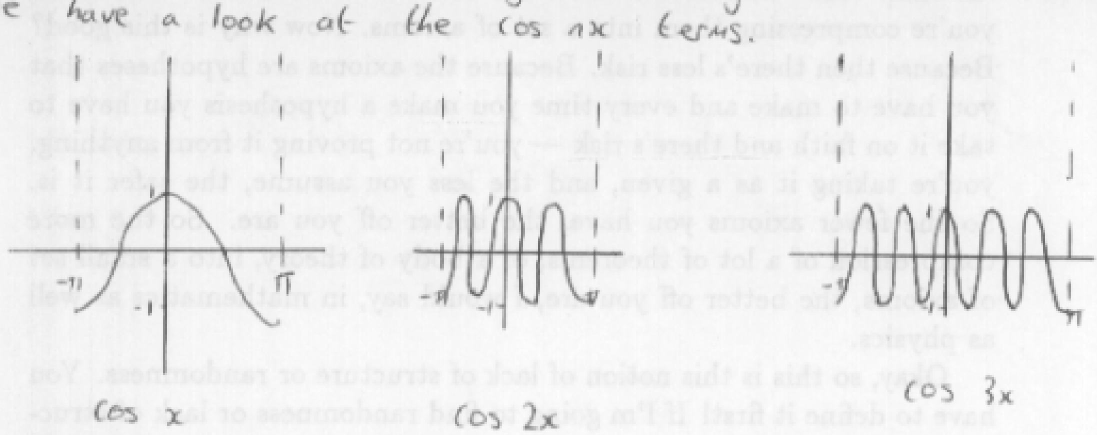
$$F(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \ldots + b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \ldots$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

The important question is how do we calculate the values of the constants $a_0, a_1, a_2, \ldots b_1, b_2, \ldots$. We will start by integrating $F(x)$

$$\int_{-\pi}^{\pi} F(x)\,dx = \int_{-\pi}^{\pi} \frac{a_0}{2}\,dx + \sum_{n=1}^{\infty} a_n \int_{-\pi}^{\pi} \cos nx\,dx + \sum_{n=1}^{\infty} b_n \int_{-\pi}^{\pi} \sin nx\,dx$$

Integrating any function is a way of finding the area beneath the function. If we have a look at the $\cos nx$ terms.



When the function is above the x-axis, the area between the x axis and the function counts as positive area. When it's below the x-axis, it counts as negative area. In all of the $\cos nx$ cases, there is the same area above and below the curve, so the integral is zero.

The same is also true of all the sin integrals, which means that