So we have $a=0$ $b=-c$. We are free to choose either b or c.

If we choose $c=1$, then $b=-1$ and we have $\begin{pmatrix}0\\-1\\1\end{pmatrix}$

If $\lambda=1$ $\begin{pmatrix}1&0&0\\0&1&1\\0&1&1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}a\\b\\c\end{pmatrix}$ $\quad a=a$, $b+c=b$, $b+c=c$. The second and third lines tell us that $b=0$ and $c=0$. We are free to choose a.

choosing $a=1$ gives $\begin{pmatrix}1\\0\\0\end{pmatrix}$.

If $\lambda=2$ $\begin{pmatrix}1&0&0\\0&1&1\\0&1&1\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=2\begin{pmatrix}a\\b\\c\end{pmatrix}$ $\quad a=2a$, $b+c=2b$, $b+c=2c$. The first line gives $a=0$. the second and third both tell us that $b=c$.

Choosing $c=1$ gives $\begin{pmatrix}0\\1\\1\end{pmatrix}$

$T=\begin{pmatrix}0&1&0\\-1&0&1\\1&0&1\end{pmatrix}$ $\quad \det(T)=-1(-1(1)-1)=2\neq 0$

$\therefore$ T is invertible

D) $D=\begin{pmatrix}2&0&-2\\0&3&0\\0&0&3\end{pmatrix}$ $\quad |D-\lambda I|=0$ $\quad \begin{vmatrix}2-\lambda&0&-2\\0&3-\lambda&0\\0&0&3-\lambda\end{vmatrix}=0$

$(2-\lambda)(3-\lambda)(3-\lambda)-2(0)=0$ $\quad \lambda=2,3,3$

If $\lambda=2$.

$\begin{pmatrix}2&0&-2\\0&3&0\\0&0&3\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=2\begin{pmatrix}a\\b\\c\end{pmatrix}$ $\quad 2a-2c=2a$, $3b=2b$, $3c=2c$. The second and third lines give $b=0$ and $c=0$. We are free to choose a.

Choosing $a=1$ gives $\begin{pmatrix}1\\0\\0\end{pmatrix}$

If $\lambda=3$

$\begin{pmatrix}2&0&-2\\0&3&0\\0&0&3\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=3\begin{pmatrix}a\\b\\c\end{pmatrix}$ $\quad 2a-2c=3a$, $3b=3b$, $3c=3c$. Rearranging the first line gives $a=-2c$. However there are no constraints on c or b

choosing $b=1$ and $c=1$ gives $\begin{pmatrix}-2\\1\\1\end{pmatrix}$

For the third eigenvector we choose values of c and b that give an eigenvector that is linearly independent of the other two.

$b=-1$ and $c=1$ gives $\begin{pmatrix}-2\\-1\\1\end{pmatrix}$ which is independent of $\begin{pmatrix}1\\0\\0\end{pmatrix}$ and $\begin{pmatrix}-2\\1\\1\end{pmatrix}$.

T is $\begin{pmatrix}1&-2&-2\\0&1&-1\\0&1&1\end{pmatrix}$ $\quad \det(T)=1(1+1)+2(0)-2(0)=2\neq 0$

$\therefore$ T is invertible.