So $\begin{pmatrix} \frac{3}{4} \\ 1 \end{pmatrix}$

For $\lambda = 1$ $\quad \begin{pmatrix} -14 & 12 \\ -20 & 17 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ $\qquad \begin{matrix} -14a + 12b = a \\ -20a + 17b = b \end{matrix}$

The First line gives $12b = 15a$ or $a = \frac{4}{5}b$. The second line gives the same. Choosing $b=1$, a must be $\frac{4}{5}$, so we have $\begin{pmatrix} \frac{4}{5} \\ 1 \end{pmatrix}$

$T = \begin{pmatrix} \frac{3}{4} & \frac{4}{5} \\ 1 & 1 \end{pmatrix}$. $\quad \det(T) = \frac{3}{4} - \frac{4}{5} \neq 0$, so T is the invertible matrix referred to in the question.

B) $\quad B = \begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}$ $\quad |B - \lambda I| = 0$ $\quad \begin{vmatrix} 1-\lambda & 0 \\ 6 & -1-\lambda \end{vmatrix} = 0$

$(1-\lambda)(-1-\lambda) = 0$ $\quad \lambda = 1$ or -1

If $\lambda = 1$

$\begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ b \end{pmatrix}$ $\quad \begin{matrix} a = a \\ 6a - b = b \end{matrix}$

From the second line $6a = 2b$ $\quad a = \frac{1}{3}b$. Choosing $b=1$, a must be $\frac{1}{3}$ giving $\begin{pmatrix} \frac{1}{3} \\ 1 \end{pmatrix}$

If $\lambda = -1$

$\begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -\begin{pmatrix} a \\ b \end{pmatrix}$ $\quad \begin{matrix} a = -a \\ 6a - b = -b \end{matrix}$ $\quad$ The first line tells us that $a = 0$. However, there's no constraint on b.

Choosing $b = 1$ gives $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

With these eigenvectors, $T = \begin{pmatrix} \frac{1}{3} & 0 \\ 1 & 1 \end{pmatrix}$. $\quad \det(T) = \frac{1}{3} \neq 0$. so T is invertible.

c) $\quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ $\quad |C - \lambda I| = 0$ $\quad \begin{vmatrix} 1-\lambda & 0 & 0 \\ 0 & 1-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda)\left((1-\lambda)(1-\lambda) - 1\right) = 0$

$(1-\lambda)(1 - 2\lambda + \lambda^2 - 1) = 0$

$(1-\lambda)(\lambda^2 - 2\lambda) = 0$

$\lambda(1-\lambda)(\lambda - 2) = 0$

so $\lambda = 0, 1, 2$.

If $\lambda = 0$ $\quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0$ $\quad \begin{matrix} a = 0 \\ b + c = 0 \\ b + c = 0 \end{matrix}$