So, For $\lambda=3$ we have $\begin{pmatrix}1\\0\\0\end{pmatrix}$.

For $\lambda=2$
$$\begin{pmatrix}3&0&0\\0&2&0\\0&1&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=2\begin{pmatrix}a\\b\\c\end{pmatrix} \qquad \begin{aligned}3a&=2a\\2b&=2b\\b+2c&=2c\end{aligned}$$

The First line gives $a=0$. The third line gives $b=0$. There's no constraint on c so we choose $c=1$. The eigenvector is $\begin{pmatrix}0\\0\\1\end{pmatrix}$.

There are two eigenvalues of 2, so we choose another value of c, as part of a third eigenvector. Taking $c=2$ gives $\begin{pmatrix}0\\0\\2\end{pmatrix}$

The three eigenvectors are $\begin{pmatrix}1\\0\\0\end{pmatrix}$, $\begin{pmatrix}0\\0\\1\end{pmatrix}$ and $\begin{pmatrix}0\\0\\2\end{pmatrix}$. Using these for the columns of T gives.

$$T=\begin{pmatrix}1&0&0\\0&0&0\\0&1&2\end{pmatrix} \qquad \det(T)=0 \quad \text{so } B \text{ cannot be diagonalized.}$$

Question 2

In each of the Following cases, Find an invertible matrix that diagonalizes the given matrix.

$$A=\begin{pmatrix}-14&12\\-20&17\end{pmatrix} \quad B=\begin{pmatrix}1&0\\6&-1\end{pmatrix} \quad C=\begin{pmatrix}1&0&0\\0&1&1\\0&1&1\end{pmatrix} \quad D=\begin{pmatrix}2&0&-2\\0&3&0\\0&0&3\end{pmatrix}$$

Solution

A)
$$\begin{vmatrix}-14-\lambda&12\\-20&17-\lambda\end{vmatrix}=0 \qquad (-14-\lambda)(17-\lambda)+(12)(20)=0$$
$$(-238-17\lambda+14\lambda+\lambda^2+240)=0$$
$$\lambda^2-3\lambda+2=0$$
$$(\lambda-2)(\lambda-1)=0$$
$$\lambda=2,1$$

To get the eigenvectors:

For $\lambda=2$
$$\begin{pmatrix}-14&12\\-20&17\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=2\begin{pmatrix}a\\b\end{pmatrix} \quad \text{This gives} \quad \begin{aligned}-14a+12b&=2a\\-20a+17b&=2b\end{aligned}$$

Rearranging the First line : $12b=16a$ $\quad a=\frac{3}{4}b$.

Rearranging the second line : $15b=20a$ $\quad a=\frac{3}{4}b$.

We are Free to choose one component and then we can calculate the other From this. Choosing $b=1$, a must be $\frac{3}{4}$.