A can be diagonalized. If the determinant is zero, T^{-1} does not exist and A cannot be diagonalized.

One of the reasons that diagonalizing matrices is useful is that, when a matrix is successfully diagonalized, the nonzero elements are the eigenvalues of the matrix.

Let's calculate the eigenvalues of A and B.

$A = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}$ $\quad \lambda = 2, 2.$

$\begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 2\begin{pmatrix} a \\ b \end{pmatrix}$ we need to solve this for a & b.

$2a = 2a$ From the top row of the matrix equation

$a + 2b = 2b$ From the bottom row.

Rearranging the second line gives. $a = 0$.

Substituting this in to the second line leaves $2b = 2b$, which tells us nothing except that there is no constraint on the value of b.

Consequently, we can choose b to take any value.

We take $b = 1$ to give $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ as one eigenvector.

A is a 2x2 matrix and we have 2 eigenvalues, so we need a second eigenvector. Because both eigenvalues are the same, the equations for the second eigenvector are the same: $a = 0$, $2b = 2b$.

So, we choose another value for b, arbitrarily. let's choose 2.

The eigenvalues and eigenvectors are

$\lambda = 2 \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \lambda = 2 \quad \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

Can we diagonalize A? To construct T, we use $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 2 \end{pmatrix}$ as the columns. $T = \begin{pmatrix} 0 & 0 \\ 1 & 2 \end{pmatrix}$ Determinant$(T) = \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} = 0$

This means that T^{-1} does not exist and that A cannot be diagonalized.

To work out the eigenvalues of B

$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 3\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ so $\begin{aligned} 3a &= 3a \\ 2b &= 3b \\ b + 2c &= 3c \end{aligned}$

Firstly, we use the second line $2b = 3b$. Rearranging this gives $b = 0$. Substituting for b in the third line gives $2c = 3c$, which simplifies to $c = 0$. The first line, $3a = 3a$, doesn't restrict a, so we are free to give it an arbitrary value. Let's choose $a = 1$.