which is the same as $\begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 0.$

Question 1

Find the eigenvalues and corresponding eigenvectors of the following matrices

$$A = \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

and find out if they can be diagonalized.

Solution

We need to solve $|A - \lambda I| = 0$ for λ to get the eigen values.

$$\left| \begin{pmatrix} 2 & 0 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = \begin{vmatrix} 2-\lambda & 0 \\ 1 & 2-\lambda \end{vmatrix} = (2-\lambda)(2-\lambda) = 0$$

$\lambda = 2$ and 2.

For B,

$$\left| \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \right| = \begin{vmatrix} 3-\lambda & 0 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 1 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda)(2-\lambda)$$

$\lambda = 3, 2$ and 2.

Before we can do the second part, we need a bit more theory.

Some matrices can be transformed so that the only elements that are non-zero are those on the diagonal that runs from the top left to the bottom right. To transform a matrix A, we need another matrix T which is formed by putting the eigenvectors in each column.

A can be transformed into a diagonal matrix D with the following formula $D = T^{-1}AT$.

Not all matrices can be diagonalized and if A is a matrix that can't be diagonalized, then T will not have an inverse. The easiest way to find out if A can be diagonalized is to calculate the determinent of T. If it is non zero, T^{-1} exists and