Eigenvalues and Eigenvectors.

The word 'Eigen' is German for 'characteristic'. Eigen values and eigenvectors are quantities that we obtain from matrices and they can be used to demonstrate certain properties of the matrices, which is why they are 'characteristic'.

The number of eigenvalues and eigen vectors that a matrix has is dependent on the dimension of the matrix. A 2x2 matrix has 2 eigenvalues and 2 eigenvectors. A 3x3 matrix has 3 of each. A nxn has n of each. The eigenvalues may not all be different, but the eigenvectors are.

To obtain the eigen values of matrix A, we subtract λI (where I is the identity matrix) from A, calculate the determinent and set it equal to zero. This gives us an equation for λ. The solutions are the eigen values.

For example, if $A = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ $\quad A - \lambda I = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{pmatrix}$

$|A - \lambda I| = (2-\lambda)(2-\lambda) - 1 = 4 - 4\lambda + \lambda^2 - 1 = \lambda^2 - 4\lambda + 3$

setting this equal to zero $\lambda^2 - 4\lambda + 3 = 0 \Rightarrow (\lambda - 3)(\lambda - 1) = 0$

The eigen values are 3 and 1

One of the useful properties of eigenvectors is that when one is multiplied by A, the result is the same as the eigenvector multiplied by the appropriate eigenvalue. In this way, the eigenvectors and values pair off. The i^{th} eigenvector, multiplied by A, is the same as the i^{th} eigenvector, multiplied by the i^{th} eigen value.

If $\underline{v}_i$ is the i^{th} eigenvector and λ_i is the i^{th} eigen value,

$$A\underline{v}_i = \lambda_i \underline{v}_i$$

We can use this property to calculate the eigenvectors. For the example above $\underline{v}_i$ must have two components. Taking $\lambda = 3$