$\int_{-\pi}^{\pi} 1 \, d\theta = 2\pi$. So we replace 1 with $\frac{1}{\sqrt{2\pi}}$

$\int_{-\pi}^{\pi} \cos\theta \cos\theta \, d\theta = \pi$, so we replace $\cos\theta$ with $\frac{1}{\sqrt{\pi}}\cos\theta$

$\int_{-\pi}^{\pi} \sin\theta \sin\theta \, d\theta = \pi$, so we replace $\sin\theta$ with $\frac{1}{\sqrt{\pi}}\sin\theta$

$\int_{-\pi}^{\pi} \cos 2\theta \cos 2\theta \, d\theta = \pi$, so we replace $\cos 2\theta$ with $\frac{1}{\sqrt{\pi}}\cos 2\theta$

$\int_{-\pi}^{\pi} \sin 2\theta \sin 2\theta \, d\theta = \pi$, so we replace $\sin 2\theta$ with $\frac{1}{\sqrt{\pi}}\sin 2\theta$.

The orthonormalized set is $\left\{\frac{1}{\sqrt{2\pi}}, \frac{1}{\sqrt{\pi}}\cos\theta, \frac{1}{\sqrt{\pi}}\sin\theta, \frac{1}{\sqrt{\pi}}\cos 2\theta, \frac{1}{\sqrt{\pi}}\sin 2\theta\right\}$