Product between 1 and 1: $\int_{-\pi}^{\pi} 1.1\, d\theta = [\theta]_{-\pi}^{\pi} = 2\pi$

$1 \;\&\; \cos\theta$: $\int_{-\pi}^{\pi} 1.\cos\theta\, d\theta = \int_{-\pi}^{\pi} \cos\theta\, d\theta = [\sin\theta]_{-\pi}^{\pi} = 0$

$1 \;\&\; \sin\theta$: $\int_{-\pi}^{\pi} 1.\sin\theta\, d\theta = \int_{-\pi}^{\pi} \sin\theta\, d\theta = [-\cos\theta]_{-\pi}^{\pi} = 1 - 1 = 0$

$1 \;\&\; \cos 2\theta$: $\int_{-\pi}^{\pi} 1.\cos 2\theta\, d\theta = \int_{-\pi}^{\pi} \cos 2\theta\, d\theta = [\tfrac{1}{2}\sin 2\theta]_{-\pi}^{\pi} = 0$

$1 \;\&\; \sin 2\theta$: $\int_{-\pi}^{\pi} 1.\sin 2\theta\, d\theta = \int_{-\pi}^{\pi} \sin 2\theta\, d\theta = [-\tfrac{1}{2}\cos 2\theta]_{-\pi}^{\pi} = -\tfrac{1}{2} + \tfrac{1}{2} = 0$

$\cos\theta \;\&\; \cos\theta$: $\int_{-\pi}^{\pi} \cos\theta\cos\theta\, d\theta = \int_{-\pi}^{\pi} \cos^2\theta\, d\theta = \int_{-\pi}^{\pi} \tfrac{1}{2}(1+\cos 2\theta)\, d\theta = \tfrac{1}{2}[\theta + \tfrac{1}{2}\sin 2\theta]_{-\pi}^{\pi} = \tfrac{\pi}{2} + \tfrac{\pi}{2} = \pi$

$\cos\theta \;\&\; \sin\theta$: $\int_{-\pi}^{\pi} \cos\theta\sin\theta\, d\theta = \int_{-\pi}^{\pi} \tfrac{1}{2}\sin 2\theta\, d\theta = [-\tfrac{1}{4}\cos 2\theta]_{-\pi}^{\pi} = -\tfrac{1}{4} + \tfrac{1}{4} = 0.$

$\cos\theta \;\&\; \cos 2\theta$: $\int_{-\pi}^{\pi} \cos\theta\cos 2\theta\, d\theta = [\sin\theta\cos 2\theta]_{-\pi}^{\pi} + \int_{-\pi}^{\pi} 2\sin\theta\sin 2\theta\, d\theta = \int_{-\pi}^{\pi} 2\sin\theta\sin 2\theta\, d\theta = [-2\cos\theta\sin 2\theta]_{-\pi}^{\pi}$

$-\int_{-\pi}^{\pi} -4\cos\theta\cos 2\theta\, d\theta = \int_{-\pi}^{\pi} 4\cos\theta\cos 2\theta\, d\theta$. Using integration by parts.

So $\int_{-\pi}^{\pi} \cos\theta\cos 2\theta\, d\theta = \int_{-\pi}^{\pi} 4\cos\theta\cos 2\theta\, d\theta \Rightarrow 3\int_{-\pi}^{\pi} \cos\theta\cos 2\theta\, d\theta = 0.$

$\cos\theta \;\&\; \sin 2\theta$: $\int_{-\pi}^{\pi} \cos\theta\sin 2\theta\, d\theta = [\sin\theta\sin 2\theta]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} 2\sin\theta\cos 2\theta\, d\theta = -\int_{-\pi}^{\pi} 2\sin\theta\cos 2\theta\, d\theta = [2\cos\theta\cos 2\theta]_{-\pi}^{\pi}$

$-\int_{-\pi}^{\pi} -4\cos\theta\sin 2\theta\, d\theta = \int_{-\pi}^{\pi} 4\cos\theta\sin 2\theta\, d\theta \Rightarrow 3\int_{-\pi}^{\pi} \cos\theta\sin 2\theta\, d\theta = 0.$

$\sin\theta \;\&\; \sin\theta$: $\int_{-\pi}^{\pi} \sin\theta\sin\theta\, d\theta = \int_{-\pi}^{\pi} \sin^2\theta\, d\theta = \int_{-\pi}^{\pi} \tfrac{1}{2}(1-\cos 2\theta)\, d\theta = \tfrac{1}{2}[\theta - \tfrac{1}{2}\sin 2\theta]_{-\pi}^{\pi} = \pi$

$\sin\theta \;\&\; \cos 2\theta$: $\int_{-\pi}^{\pi} \sin\theta\cos 2\theta\, d\theta = [-\cos\theta\cos 2\theta]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} 2\cos\theta\sin 2\theta\, d\theta = -\int_{-\pi}^{\pi} 2\cos\theta\sin 2\theta\, d\theta$

$= -[2\sin\theta\sin 2\theta]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} -4\sin\theta\cos 2\theta\, d\theta = \int_{-\pi}^{\pi} 4\sin\theta\cos 2\theta\, d\theta \Rightarrow 3\int_{-\pi}^{\pi} \sin\theta\cos 2\theta\, d\theta = 0$

$\sin\theta \;\&\; \sin 2\theta$: $\int_{-\pi}^{\pi} \sin\theta\sin 2\theta\, d\theta = -[\cos\theta\sin 2\theta]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} -2\cos\theta\cos 2\theta\, d\theta = \int_{-\pi}^{\pi} 2\cos\theta\cos 2\theta\, d\theta$

$= [2\sin\theta\cos 2\theta]_{-\pi}^{\pi} - \int_{-\pi}^{\pi} -4\sin\theta\sin 2\theta\, d\theta = \int_{-\pi}^{\pi} 4\sin\theta\sin 2\theta\, d\theta.$

$\Rightarrow \int_{-\pi}^{\pi} 3\sin\theta\sin 2\theta\, d\theta = 0.$

$\cos 2\theta \;\&\; \cos 2\theta$: $\int_{-\pi}^{\pi} \cos 2\theta\cos 2\theta\, d\theta = \int_{-\pi}^{\pi} \cos^2 2\theta\, d\theta = \int_{-\pi}^{\pi} \tfrac{1}{2}(1+\cos 4\theta)\, d\theta = \tfrac{1}{2}[\theta + \tfrac{1}{4}\sin 4\theta]_{-\pi}^{\pi} = \pi$

$\cos 2\theta \;\&\; \sin 2\theta$: $\int_{-\pi}^{\pi} \cos 2\theta\sin 2\theta\, d\theta = \tfrac{1}{2}\int_{-\pi}^{\pi} \sin 4\theta\, d\theta = [-\tfrac{1}{8}\cos 4\theta]_{-\pi}^{\pi} = -\tfrac{1}{8} + \tfrac{1}{8} = 0$

$\sin 2\theta \;\&\; \sin 2\theta$: $\int_{-\pi}^{\pi} \sin 2\theta\sin 2\theta\, d\theta = \int_{-\pi}^{\pi} \sin^2 2\theta\, d\theta = \int_{-\pi}^{\pi} \tfrac{1}{2}(1-\cos 4\theta)\, d\theta = \tfrac{1}{2}[\theta - \tfrac{1}{4}\sin 4\theta]_{-\pi}^{\pi}$

$= \pi$

The product is only non-zero when the two functions are the same. Therefore the set is orthogonal.

To make the set orthonormal, we need to modify the functions so that when the product is taken between a function and itself, the result is 1.