Tutorial 17

The inner product

So far, we have looked at subspaces and seen the criteria that define them. We have also looked at vectors and the definition of linear independence. We saw that we could apply these ideas to more than just vectors; they were also applicable to matrices and functions.

In this tutorial, we are going to look at another important part of the vector picture and see how the same idea can be applied elsewhere. We are going to look at the product.

For two 3 dimensional vectors, $\underline{u}$ and $\underline{v}$, we are used to seeing the dot product between them. $\underline{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

$\underline{u} \cdot \underline{v} = u_1 v_1 + u_2 v_2 + u_3 v_3.$

This is also equal to $\|\underline{u}\| \, \|\underline{v}\| \cos\theta$, where $\|\underline{u}\|$ is just the length of $\underline{u}$ and $\|\underline{v}\|$ the length of $\underline{v}$. θ is the angle between the two vectors. It's useful because it gives us the product of $\|\underline{u}\|$ and $\|\underline{v}\|$ in a common direction. However, essentially it is just a way of using two vectors to obtain a scalar number.

There are many different ways of using vectors to obtain a scalar number. They are called products. Some are more useful than others. The useful ones are called 'inner products' and satisfy the criteria below. The 'dot product' is an example of an inner product. We will denote the product between the vectors $\underline{u}$ and $\underline{v}$ by $\langle \underline{u}, \underline{v} \rangle$.

For a product to be an inner product -

a) It must not matter which way round we take the product, ie $\langle \underline{u}, \underline{v} \rangle = \langle \underline{v}, \underline{u} \rangle$.

b) The product of a vector with itself must be greater than or equal to zero, ie $\langle \underline{u}, \underline{u} \rangle \geq 0$.

c) It must not matter whether we multiply by a scalar before or after we take the product, ie. if α is a scalar $\langle \underline{u}, \alpha\underline{v} \rangle = \alpha \langle \underline{u}, \underline{v} \rangle$

d) It must not matter whether we add vectors before or after we take the product, ie if $\underline{w}$ is a 3rd vector $\langle \underline{u}, \underline{v}+\underline{w} \rangle = \langle \underline{u}, \underline{v} \rangle + \langle \underline{u}, \underline{w} \rangle$.