rearranging ② $q = -\frac{1}{2}$.

Finally, substituting for q in ① gives us $0 = p$.

So our coordinates in the new basis are $(0, -\frac{1}{2}, \frac{1}{3})$.

We can check that we have not made any mistakes, by substituting for the values of p, q and r in the equation

$$\begin{pmatrix}0\\0\\1\end{pmatrix} = p\begin{pmatrix}1\\0\\0\end{pmatrix} + q\begin{pmatrix}2\\2\\0\end{pmatrix} + r\begin{pmatrix}3\\3\\3\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\1\end{pmatrix} = 0\begin{pmatrix}1\\0\\0\end{pmatrix} - \frac{1}{2}\begin{pmatrix}2\\2\\0\end{pmatrix} + \frac{1}{3}\begin{pmatrix}3\\3\\3\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} - \begin{pmatrix}1\\1\\0\end{pmatrix} + \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\1\end{pmatrix} \text{ as required.}$$

For a general vector $\begin{pmatrix}a\\b\\c\end{pmatrix}$

$$\begin{pmatrix}a\\b\\c\end{pmatrix} = p\begin{pmatrix}1\\0\\0\end{pmatrix} + q\begin{pmatrix}2\\2\\0\end{pmatrix} + r\begin{pmatrix}3\\3\\3\end{pmatrix}$$

① $a = p + 2q + 3r$
② $b = 2q + 3r$
③ $c = 3r$

③ Gives $r = \frac{c}{3}$. Substituting for r in ② and ①.

① $a = p + 2q + c$
② $b = 2q + c$

Rearranging ②: $q = \frac{1}{2}(b-c)$. Substituting for q in ①

$a = p + (b-c) + c = p + b \qquad p = a - b$.

Hence, the coordinates are $(a-b, \frac{1}{2}(b-c), \frac{c}{3})$

Question 3

Calculate the length of the following curves

a) $y = \frac{1}{3}(x^2+2)^{3/2}$ from $x=0$ to $x=3$

b) $r = 1$ in polar coordinates from $\theta = 0$ to $\theta = 2\pi$

c) $x = \frac{y^4}{4} + \frac{1}{8y^2}$ from $y=1$ to $y=2$

Solution

This takes us back to the material on line integrals.

a) $y = \frac{1}{3}(x^2+2)^{3/2}$ We want $\int_{x=0}^{x=3} ds$ where $ds = \sqrt{dx^2+dy^2}$

$ds = \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$. $\int_{x=0}^{x=3} ds = \int_0^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$