The second column is twice the First, so all three columns are not linearly independent. However, the second and third columns are independent, so the rank is 2.

We can calculate this with the determinent as Follows. Firstly, we take the determinent of the whole matrix. It's equal to zero, so the rank cannot be 3. Next, to see if it is 2, we look at 2×2 sections of the matrix.

The determinent of $\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$, which is the top left 2×2 section, is zero, as is the determinent of $\begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$. Looking at the top right corner, $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ has a zero determinent. However $\begin{pmatrix} 4 & 6 \\ 6 & 10 \end{pmatrix}$, From the bottom right corner does not. We must also look at the submatrices Found by rearranging the order of the columns. Swapping the 1st and 2nd columns and taking the top right and bottom right submatrices, we have $\begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$ and $\begin{pmatrix} 2 & 6 \\ 3 & 10 \end{pmatrix}$. The Former has a zero determinent, but the latter does not.

Now the largest submatrix that has had a non zero determinent has been 2×2. Hence the rank must be 2.

If all the 2×2 sections had had zero determinents, we would have to have resorted to 1×1 'submatrices', the determinent of which is equal to the value of the matrix element that makes up the 1×1 'submatrix'.

Question 1

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 2 & 5 & 3 \\ 3 & 6 & 3 \end{pmatrix} \qquad B = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Find the ranks of the above two matrices.

Solution

A) We can see that the 2nd column is equal to the 1st + 3rd columns, so the columns aren't linearly independent. Hence, the rank is not 3. A calculation of the determinent gives 0, confirming this. However, the First and second columns are linearly independent, so the rank must be 2. The determinent of the 'submatrix' $\begin{pmatrix} 1 & 4 \\ 2 & 5 \end{pmatrix}$ is non-zero, confirming this.