we make the expression, for the total length of the curve, exact. $u \rightarrow dx$. $v \rightarrow dy$ $s \rightarrow ds$ $\sum_i \rightarrow \int$

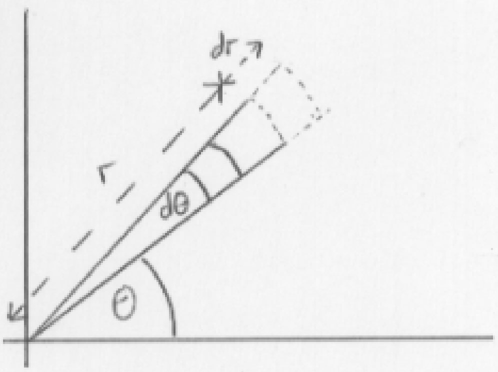
$\sum_i \sqrt{u_i^2 + v_i^2} \rightarrow \int \sqrt{dx^2+dy^2}$ where $ds^2 = dx^2+dy^2$.

The expression $\int \sqrt{dx^2+dy^2}$ is difficult to evaluate. However, if we take out a factor of dx, it becomes more familiar; $\int \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$.

We have the equation of the curve, $y(x)$, so we can calculate $\frac{dy}{dx}$ and $\left(\frac{dy}{dx}\right)^2$ to give us the expression inside the square root. Hence, the solution is now just an integral of $\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$.

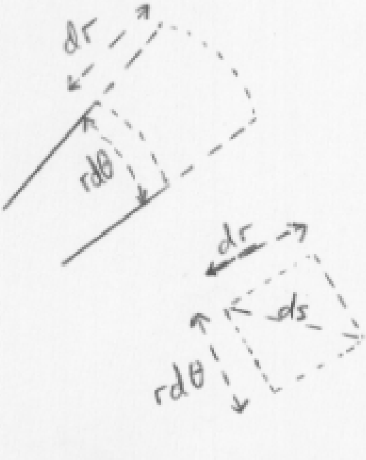
In some cases, we may not have the equation of the curve as a function of y in terms of x. We may have it as a function of x in terms of y, $x(y)$. Taking the expression $\int \sqrt{dx^2+dy^2}$, instead of moving dx outside the square root, we move dy, to give us $\int \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$. Because we know $x(y)$, we can calculate $\frac{dx}{dy}$ by differentiating $x(y)$ and the question will give us the limits on the y-integral. Hence, when we have $x(y)$, the solution is an integral in y.

In polar coordinates, it looks slightly different.



When we looked at polar coordinates, we saw that the area bounded by the dotted lines became square, when $d\theta$ and dr became infinitely small. We also saw that one side of the square was given by $r\,d\theta$ and that the other side was given by dr. So the area of the square was $r\,d\theta\,dr$.

In this case, $ds^2 = r^2 d\theta^2 + dr^2$.



Now, in polar coordinates, curves are described by equations of the type $r(\theta)$. $r(\theta)$ relates, for each point on the curve, its distance from the origin and its direction relative to the x-axis