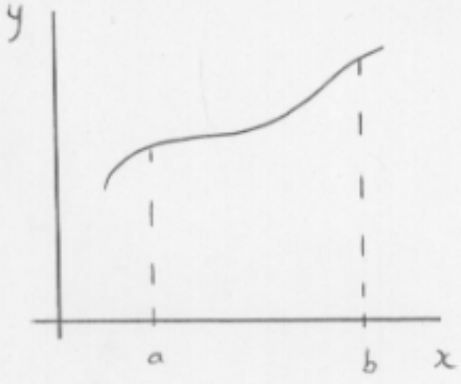
Tutorial 15

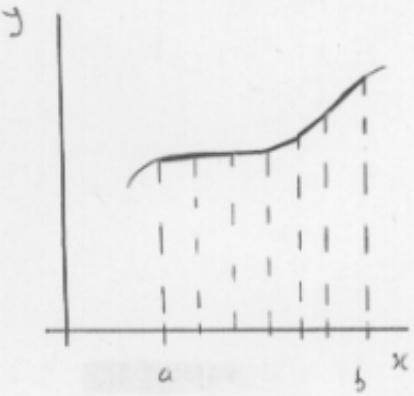
Line integrals

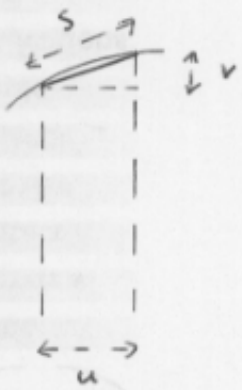
We have seen how to use integrals to calculate the volume and the area, with 3-dimensional integrals and 2-dimensional integrals. However, we have yet to use integrals to calculate the length of curves, which is the 1 dimensional analogue. This is what we shall cover in this tutorial.

Suppose that we have a Function, $y(x)$, and that we want to Find the length of the curve between the points $x=a$ and $x=b$.



A crude way to approximate the length would be to divide the curve up in to sections, approximate each section with a straight line, work out the length of the straight lines and add this up over all the sections.





This is what one section looks like.
s is the length of the straight line that approximates the curve.
By pythagoras $s^2 = u^2 + v^2$.

The approximate value of the total length of the curve is $\sum_i s_i$, where i runs over all the sections. Using pythagoras, this is equal to $\sum_i \sqrt{u_i^2 + v_i^2}$

Now let's make u, v and s infinitely small and in doing so