$$\alpha\begin{pmatrix}\lambda\\-1/2\\-1/2\end{pmatrix}+\beta\begin{pmatrix}-1/2\\\lambda\\-1/2\end{pmatrix}+\gamma\begin{pmatrix}-1/2\\-1/2\\\lambda\end{pmatrix}=0$$

This gives us 3 simultaneous equations.

(1) $\alpha\lambda - \frac{1}{2}\beta - \frac{1}{2}\gamma = 0$

(2) $-\frac{1}{2}\alpha + \lambda\beta - \frac{1}{2}\gamma = 0$

(3) $-\frac{1}{2}\alpha - \frac{1}{2}\beta + \lambda\gamma = 0$

Rearranging (1) gives $\gamma = 2\alpha\lambda - \beta$. Substituting for γ in (2) & (3) gives

(2) $-\frac{1}{2}\alpha + \lambda\beta - \alpha\lambda + \frac{1}{2}\beta = 0$

(3) $-\frac{1}{2}\alpha - \frac{1}{2}\beta + 2\alpha\lambda^2 - \lambda\beta = 0$

rearranging (2) $\alpha(\frac{1}{2}+\lambda) = \beta(\frac{1}{2}+\lambda)$ $\therefore \alpha = \beta$

rearranging (3) $(-\frac{1}{2} - \frac{1}{2} + 2\lambda^2 - \lambda)\alpha = 0$

$2\lambda^2 - \lambda - 1 = 0$

$(2\lambda+1)(\lambda-1) = 0$

$\lambda = 1$ or $-1/2$.

when $\lambda = -1/2$ clearly $\underline{V}_1, \underline{V}_2, \underline{V}_3$ are not linearly independent.
Hence, $\lambda = 1$.

We now have 3 linearly independent vectors in a 3-dimensional space. So they can be used as a basis for the space.