Can we construct 6 from $3\sin^2 x$ and $2\cos^2 x$?

$2(3\sin^2 x) + 3(2\cos^2 x) = 6\sin^2 x + 6\cos^2 x = 6$. So, yes we can and the set $6, 3\sin^2 x, 2\cos^2 x$ are linearly dependent.

b) Can we construct x as a linear combination of $\cos x$? No. We can see this if we plot $F(x) = x$ and $F(x) = \cos x$. Clearly we can't obtain a straight line plot by multiplying a constant and an oscillating (ie wave-like) function. Linearly independent

c) Can we obtain 1 from $\sin x$ and $\sin 2x$? No. If we plot $F(x) =$, $F(x) = \sin x$ and $F(x) = \sin 2x$, we can see that we can't construct $\sin x$ from a linear combination of a straight line function $F(x) = x$ and an oscillating function of a different frequency, $F(x) = \sin 2x$. Linearly independent

d) Can we obtain $\cos 2x$ from $\sin^2 x$ and $\cos^2 x$? Yes, but we need the trig identities $\cos^2 x + \sin^2 x = 1$ and $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$.

$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$. $\qquad 2\cos^2 x = (1 + \cos 2x)$

$2\cos^2 x - (\cos^2 x + \sin^2 x) = \cos 2x$

$2\cos^2 x - (\cos^2 x + \sin^2 x) = \cos^2 x - \sin^2 x$

So $\cos 2x = \cos^2 x - \sin^2 x$. The set is linearly dependent

e) Can we construct $(3-x)^2$ from $x^2 - 6x$ and 5?

$(3-x)^2 = 9 - 6x + x^2 = (x^2 - 6x) + \frac{9}{5}(5)$. So $(3-x)^2$ can be constructed as a linear combination of $x^2 - 6x$ and 5. Hence, the set is linearly dependent.

F) Remember the definition that $\alpha \underline{v}_1 + \beta \underline{v}_2 + \gamma \underline{v}_3 = 0$ must have a solution with either $\alpha \neq 0$, $\beta \neq 0$ or $\gamma \neq 0$? If $\underline{v}_1 = 0$, $\underline{v}_2 = \cos^3 \pi x$, $\underline{v}_3 = \sin^3 \pi x$, then α can have any value including non-zero, so the set must be linearly dependent.

Question 4

For which values of λ are the following set of vectors, a set of linearly independent set in $\mathbb{R}^3$?

$$\underline{v}_1 = \begin{pmatrix} \lambda \\ -1/2 \\ -1/2 \end{pmatrix}, \quad \underline{v}_2 = \begin{pmatrix} -1/2 \\ \lambda \\ -1/2 \end{pmatrix}, \quad \underline{v}_3 = \begin{pmatrix} -1/2 \\ -1/2 \\ \lambda \end{pmatrix}$$

Do they form a basis for $\mathbb{R}^3$?