

- 1) For 2 real valued Functions $F(x)$ and $g(x)$, is $F(x)+g(x)$ a real valued Function? Yes.
- 2) For a real valued Function $F(x)$, is $\alpha F(x)$, where α is a scalar, a real valued Function? Yes.
- 3) For 3 real valued Functions, $F(x), g(x), h(x)$ is $(F(x)+g(x))+h(x) = F(x)+(g(x)+h(x))$? Yes.
- 4) Does there exist an identity Function, $O(x)$, so that, for a real valued Function, $F(x)$, $F(x)+O(x) = O(x)+F(x) = F(x)$?
Yes $O(x) = 0$.
- 5) For each real valued Function, $F(x)$, does there exist an inverse Function that is also real valued, so that $F(x) + \text{inverse} = O(x)$?
Yes. The inverse is $-F(x)$, which is real valued. $F(x) + (-F(x)) = O(x) = 0$.
- 6) For two real valued Functions $F(x)$ and $g(x)$, does $F(x)+g(x) = g(x)+F(x)$? Yes.
- 7) For two real valued Functions, $F(x)$ and $g(x)$, and a scalar, α , does $\alpha(F(x)+g(x)) = \alpha F(x) + \alpha g(x)$? Yes.
- 8) For a real valued Function $F(x)$ and two scalars, α and β , does $(\alpha+\beta)F(x) = \alpha F(x) + \beta F(x)$? Yes.
- 9) For a real valued Function $F(x)$ and two scalars, α and β , does $\alpha(\beta F(x)) = (\alpha\beta)F(x)$? Yes.
- 10) For a real valued Function $F(x)$, does $1 \times F(x) = F(x)$? Yes.

Hence, the set of real valued Functions, $F(x)$, is a vector space.

Question 3

Which of the following sets of Functions in $\mathbb{R}$ are linearly dependent.

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| a) $6, 3\sin^2 x, 2\cos^2 x,$ | b) $x, \cos x$ | c) $1, \sin x, \sin 2x$ |
| d) $\cos 2x, \sin^2 x, \cos^2 x$ | e) $(3-x)^2, x^2-6x, 5$ | f) $0, \cos^3 \pi x, \sin^3 \pi x$ |

Solution

$\mathbb{R}$ denotes the set of all real numbers, so a Function in $\mathbb{R}$ is one that takes one real number as a parameter and gives back a real number. A Function that takes two real numbers and gives back one, such as $F(x,y) = x+y$, could be said to be a Function from $\mathbb{R}^2$ to $\mathbb{R}$. In general, $\mathbb{R}^n$ means n independent real numbers.

It is not necessary to know this to answer the question, but it's important to understand all of the question.

- a) We need the trigonometry identity $\sin^2 x + \cos^2 x = 1$.