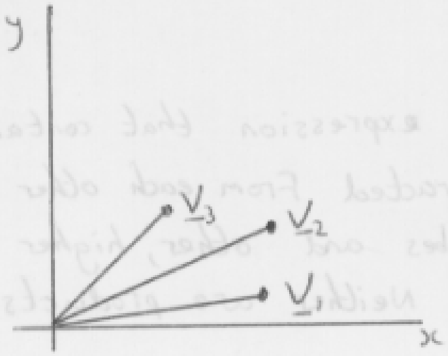
We can see this if we consider a 2-dimensional example.



$\underline{V}_1$ and $\underline{V}_2$ are linearly independent; we can see this because they aren't parallel. Because of this, we can ~~access~~ represent every point on the x-y plane as a linear combination of $\underline{V}_1$ and $\underline{V}_2$, including $\underline{V}_3$. Because we can express $\underline{V}_3$ as a linear combination of $\underline{V}_1$ and $\underline{V}_2$, the set $\underline{V}_1, \underline{V}_2, \underline{V}_3$ is not linearly independent.

Question 1

Are the following sets linearly dependent?

a) $\begin{pmatrix}-1\\2\\4\end{pmatrix}, \begin{pmatrix}5\\-10\\-20\end{pmatrix}$ b) $\begin{pmatrix}3\\-1\end{pmatrix}, \begin{pmatrix}4\\5\end{pmatrix}, \begin{pmatrix}-4\\7\end{pmatrix}$

c) $3-2x+x^2, \quad 6-4x+2x^2$ d) $A=\begin{pmatrix}-3 & 4\\2 & 0\end{pmatrix}, \quad B=\begin{pmatrix}3 & -4\\-2 & 0\end{pmatrix}$

Solution

a) Can we obtain $\begin{pmatrix}-1\\2\\4\end{pmatrix}$ as a linear combination of $\begin{pmatrix}5\\-10\\-20\end{pmatrix}$? We can because $-\frac{1}{5}\begin{pmatrix}5\\-10\\-20\end{pmatrix} = \begin{pmatrix}-1\\2\\4\end{pmatrix}$. Hence they are linearly dependent.

b) Here we have 3 vectors in a 2-dimensional space, so they must be linearly dependent.

c) Can we obtain $3-2x+x^2$ as a linear combination of $6-4x+2x^2$? We can multiply $6-4x+2x^2$ by $\frac{1}{2}$ to get $\frac{1}{2}(6-4x+2x^2)$ $= 3-2x+x^2$. Hence, they are linearly dependent.

d) Can we obtain $\begin{pmatrix}-3 & 4\\2 & 0\end{pmatrix}$ as a linear combination of $\begin{pmatrix}3 & -4\\-2 & 0\end{pmatrix}$?

Multiplying $\begin{pmatrix}3 & -4\\-2 & 0\end{pmatrix}$ by -1 gives $\begin{pmatrix}-3 & 4\\2 & 0\end{pmatrix}$. Hence, they are linearly dependent.

Question 2

Show that the set $\mathbb{R}$ of all real valued functions $f(x)$, defined on the real line, is a vector space.

Solution

To prove that it is a vector space, we use the same 10 criteria as we would to prove that it is a subspace.