not a subspace.

b) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $a+d=0$. This satisfies all 10 criteria. Hence, it is a subspace.

c) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $b+c=0$ This also satisfies all 10 criteria. Hence, it is also a subspace.

Question 3

Which of the following sets are subspaces of the polynomials of degree ≤ 3?

a) $a_0+a_1x+a_2x^2+a_3x^3$ where $a_0=0$

b) $a_0+a_1x+a_2x^2+a_3x^3$ where $a_0+a_1+a_2+a_3=0$

c) " where a_0, a_1, a_2, a_3 are all integers.

Solution

Again, we don't need to represent the polynomials as vectors. We can apply the criteria directly to the polynomials.

a) This satisfies all 10 criteria. For example, in #1, we replace the two vectors with $a_0+a_1x+a_2x^2+a_3x^3$ and $b_0+b_1x+b_2x^2+b_3x^3$

$a_0+a_1x+a_2x^2+a_3x^3+b_0+b_1x+b_2x^2+b_3x^3 = (a_0+b_0)+(a_1+b_1)x+(a_2+b_2)x^2+(a_3+b_3)x^3$

which is also of the form $C_0+C_1x+C_2x^2+C_3x^3$ where $C_0=0$.

In #2, $\frac{1}{2}(a_0+a_1x+a_2x^2+a_3x^3)$ is also of the form $C_0+C_1x+C_2x^2+C_3x^3$ with $C_0=0$.

And so The set of polynomials satisfy all 10 criteria. Hence, it is a subspace.

b) This satisfies all 10 criteria. Hence the set is a subspace.

c) This fails #2. If we multiply $a_0+a_1x+a_2x^2+a_3x^3$ (where a_0, a_1, a_2, a_3 are all integers) by $\alpha=\frac{1}{2}$, we get $\frac{a_0}{2}+\frac{a_1}{2}x+\frac{a_2}{2}x^2+\frac{a_3}{2}x^3$.

When any of a_0, a_1, a_2, a_3 are odd integers, then $\frac{a_0}{2}, \frac{a_1}{2}, \frac{a_2}{2}, \frac{a_3}{2}$ will not all be integers. Hence, $\frac{a_0}{2}+\frac{a_1}{2}x+\frac{a_2}{2}x^2+\frac{a_3}{2}x^3$ will not be of the form $C_0+C_1x+C_2x^2+C_3x^3$ (with C_0, C_1, C_2, C_3 all integer) and the set will not be a subspace.