Solution

a) $\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$ satisfies all 10 criteria. Hence, it is a subspace.

b) $\begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix}$ Fails several of the tests. It Fails #1 because $\begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} b \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a+b \\ 2 \\ 2 \end{pmatrix}$ which is not of the Form $\begin{pmatrix} c \\ 1 \\ 1 \end{pmatrix}$ (where $c = a+b$)

It also Fails #2 beause if $\alpha = \frac{1}{2}$, $\frac{1}{2}\begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a \\ 1/2 \\ 1/2 \end{pmatrix}$ which is not of the Form $\begin{pmatrix} c \\ 1 \\ 1 \end{pmatrix}$ (where $c = \frac{1}{2}a$)

c) $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ where $a = b+c$ satisfies all 10 criteria. Hence, it is a subspace.

d) $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ where $a = b+c+1$ also Fails several tests. It Fails #1 because

$\begin{pmatrix} a \\ b \\ c \end{pmatrix} + \begin{pmatrix} d \\ e \\ f \end{pmatrix}$ (with $a = b+c+1$ and $d = e+f+1$) $= \begin{pmatrix} a+d \\ b+e \\ c+f \end{pmatrix}$. However

$a+d = b+c+1+e+f+1 = (b+e)+(c+f)+2$, so it isn't of the Form $\begin{pmatrix} p \\ q \\ r \end{pmatrix}$ where $p = q+r+1$. It also Fails #2 because if

$a = b+c+1$ in $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ and we take $\alpha = \frac{1}{2}$, $\frac{1}{2}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} \frac{1}{2}a \\ \frac{1}{2}b \\ \frac{1}{2}c \end{pmatrix} = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$

Now $d = e+f+\frac{1}{2}$, which isn't the same as $d = e+f+1$. Hence, this is not a subspace.

Question 2

Which of the Following sets are subspaces of the vector space of 2×2 matrices.

a) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are integers b) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $a+d = 0$

c) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where $b+c = 0$.

Solution

We don't necessarily need to convert our matrices into vectors. We can test all 10 criteria on the matrices themselves.

a) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d integer, violates #2. If α is $\frac{1}{2}$, $\frac{1}{2}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{a}{2} & \frac{b}{2} \\ \frac{c}{2} & \frac{d}{2} \end{pmatrix}$

If any of a, b, c, d are odd integers, then $\begin{pmatrix} \frac{a}{2} & \frac{b}{2} \\ \frac{c}{2} & \frac{d}{2} \end{pmatrix}$ will not have integer elements and so don't belong to the vector space. Hence, this is