whether or not the vector space can be classified as a subspace depends on whether or not the vectors that is contains satisfy 10 criteria.

1) For any 2 vectors, $\underline{x}$ and $\underline{y}$, taken from the vector space, $x+y$ must also belong to the vector space.

2) For any vector $\underline{x}$, taken from the vector space, and a scalar, α, which can take any value, $\alpha\underline{x}$ must also belong to the vector space.

3) For any 3 vectors, $\underline{x}$, $\underline{y}$ and $\underline{z}$ taken from the vector space $(\underline{x}+\underline{y})+\underline{z} = \underline{x}+(\underline{y}+\underline{z})$

4) There must be an identity vector, called $\underline{O}$, which belongs to the vector space, so that $\underline{x}+\underline{O} = \underline{O}+\underline{x} = \underline{x}$

5) For each vector $\underline{x}$, taken from the vector space, there must exist an inverse $(-\underline{x})$, which also belongs to the vector space, so that $\underline{x}+(-\underline{x})=0$

6) For any 2 vectors, $\underline{x}$ and $\underline{y}$, taken from the vector space, $\underline{x}+\underline{y} = \underline{y}+\underline{x}$

7) For any 2 vectors, $\underline{x}$ and $\underline{y}$, taken from the vector space, and a scalar, α, which can take any value, $\alpha(\underline{x}+\underline{y}) = \alpha\underline{x}+\alpha\underline{y}$

8) For any vector $\underline{x}$, taken from the vector space and for 2 scalars, α and β, that can take any value, $(\alpha+\beta)\underline{x} = \alpha\underline{x}+\beta\underline{x}$.

9) For any vector $\underline{x}$, taken from the vector space, and for 2 scalars, α and β, that can take any value $\alpha(\beta\underline{x}) = (\alpha\beta)\underline{x}$

10) For any vector $\underline{x}$, taken from the vector space $1\underline{x} = \underline{x}$, ie one times $\underline{x}$ equals $\underline{x}$.

If all the vectors in the vector space meet all 10 criteria, then the vector space can be called a subspace.

<u>Question 1</u>

Which of the following sets are subspaces of $\mathbb{R}^3$

a) all vectors of the form $\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}$

b) " " " " $\begin{pmatrix} a \\ 1 \\ 1 \end{pmatrix}$

c) " " " " " $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ where $a=b+c$

d) " " " " " $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ where $a=b+c+1$